THE UNIVERSITY OF ALBERTA

METHODS OF REVISING INPUT-OUTPUT MATRIX PREDICTIONS WITH REFERENCE TO THE ECONOMY OF ALBERTA

by

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A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES

IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE

OF MASTER OF SCIENCE

DEPARTMENT OF AGRICULTURAL ECONOMICS AND RURAL SOCIOLOGY

EDMONTON, ALBERTA

FALL, 1970

UNIVERSITY OF ALBERTA

FACULTY OF GRADUATE STUDIES

The undersigned certify that they have read and recommend to the Faculty of Graduate Studies for acceptance a thesis entitled "Methods of Revising Input-Output Matrix Predictions with Reference to the Economy of Alberta," submitted by Hassan Fazel in partial fulfilment of the requirements for the degree of Master of Science.

ABSTRACT

This study is a theoretical exercise in the field of quantitative economic forecasting by input-output analysis. The study was undertaken on the premise that a knowledge of forecasting techniques can be helpful to economic advisers and policy makers seeking a foreknowledge of the economic environment of the future and a prevision of the future effects of their current policies.

Input-output analysis is a commonly employed technique of economic forecasting. Like all forecasts, however, input-output forecasts are liable to be inaccurate, the degree of inaccuracy increasing as forecasts are made over longer periods of time. Based on the works done at the Netherlands School of Economics and at Cambridge University, United Kingdom, this study examines some principal sources of input-output prediction errors. It is noted that the inaccuracies of economic observations and the rapid obsolescence of numerical economic and technological estimates are prime contributors to prediction errors. A mathematical investigation of the structure of prediction errors enables one to present two formulae for reducing input-output prediction errors. The two formulae are: 1) the Statistical Correction Method, which, at any point in time, utilises most recent economic data to revise original predictions in the direction of greater accuracy; 2) the RAS Method, which, in addition to most recent data, utilises various sources of knowledge of future economic and technological trends to revise original predictions.

In attempting to apply the theoretical presentations to the economy of Alberta, this study proposes an empirical input-output framework

for Alberta, with special emphasis on water-based sectors. A conceptual formulation of the input-output matrix prediction system and diagrams of the computing sequences are presented to indicate the nature of data requirements and information flows in an organised system of input-output prediction analysis.

ACKNOWLEDGEMENTS

I am grateful to Dr. W. E. Phillips for his supervision of this thesis and to Dr. L. P. Apedaile, who made useful comments on the first draft of the study.

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CHAPTER I

INTRODUCTION

Knowledge of current and future operations of an increasingly complex economy requires the employment of various analytical techniques. These techniques must be continuously refined to make them more effective aids in policy decision-making. This study examines the uses and limitations of the input-output technique of quantitative economic prediction. Specifically the study presents, in theory, the problem of input-output prediction errors and the means of reducing these errors. The theoretical results are then applied conceptually to an input-output prediction model of the economy of Alberta.

The Uses of Predictions

Predictions are of scientific interest as tentative verifications of economic models and theories. Reliable predictions are of importance for applications to economic policy.[1]

Every effort at quantitative economic forecasting presupposes a search for economic statistics. But statistics alone are not sufficient. In order to go beyond naïve modes of prediction one has to have a conception of the system of interactions to which, or within which, the forecast applies. Such a conception implies the need for some sort of

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[1]Fritz Machlup, "The Problem of Verification in Economics," Southern Economic Journal, XXII (1955), 1-21; Milton Friedman, Essays in Positive Economics (Chicago: The University of Chicago Press, 1966), p. 9; Bertrand de Jouvenel, The Art of Conjecture (New York: Basic Books, Inc., 1967), p. 84; Jacob Marschak, "Economic Structure, Path, Policy and Prediction," American Economic Review, XXXVII (May, 1947), 81-84.

theory or model of the system. In particular, estimates of the past and present economic structure, and a knowledge of expected or intended change in that structure are useful aids to public policy decision-making.

Methods of Making Forecasts

Trend extrapolation and model analysis are the two principal methods used in economic forecasting. The extrapolation of realised trends is a simple method, and it can provide some significant suggestions on the paths of development of individual economic variables. Model analysis is suitable for projections of economic structural changes. One development in the model analysis method is the growth of fully articulated forecasting systems that move down from the economy as a whole to industry and thence to the enterprise level. Such developments can be run concurrently with grass roots forecasts, which move upwards from the enterprise level. These fully articulated systems require increased knowledge of the inter-industry structure, and here the econometric model building approach may be linked with the techniques of input-output analysis.

Input-Output Analysis: Its Forecasting Uses and Limitations

The method of input-output accounting and the model of economic interdependence, which Leontief adapted from the Walrasian theory of general equilibrium, have been developed extensively in the last three decades. Current work in this field comprises three mutually interrelated, yet clearly distinct, lines of inquiry. First, there is the construction and elaboration of theoretical models and the investigation of their general properties; second, the collection and preliminary organisation of empirical data for the purpose of insertion in analytical equations; third, the manipulative and computational task of solving the equations

in order to determine specific operational properties of particular economies.

Input-output techniques can be applied to economic data for accounting uses, analytical uses, and for forecasting uses. The inverse matrix of the input-output technical coefficients matrix yields a useful device for forecasting, sector by sector, the levels of output, import requirements, or the requirements of any of the specific primary or intermediate inputs classified in the original input-output table. Open-ended input-output tables provide forecasts conditional upon the proportionality assumptions of the Leontief model and upon exogenously specified future final demands or net incomes.

A primary problem in input-output forecasting is that of prediction errors. The problem becomes even more acute when economic policy decisions are based on erroneous forecasts. Hence a preliminary analysis of prediction errors is useful. Two principal, interrelated sources of input-output prediction errors follow:

1. (a) Observational or measurement errors, which creep in when vast amounts of economic data are collected and collated for the construction of input-output tables.

   (b) Elementary prediction errors, which may be defined as the errors for a time span of one year. In various input-output experiments elementary errors have been observed to cumulate in a certain way over forecasting periods.

2. The obsolescence of numerical inter-industry coefficients contained in an input-output table, since the compilation of these coefficients for any one base year takes a long

time, during which significant changes may occur in inter-industry relationships.

Objectives of the Study and Proposed Solution

The problem considered in this study is one of dealing with the above sources of errors to revise and improve input-output matrix predictions. To improve predictions is to move in the direction of greater accuracy. Greater accuracy is important because inaccurate forecasts result in a less than optimal action; this action, in turn, leads to a utility level below an attainable maximum.

The objectives of this study are (1) a mathematical-statistical investigation of the structure of input-output prediction errors, (2) the formulation of a method of integrating more recent national accounts and inter-industry data for correcting input-output predictions, (3) a proposed inter-sectoral classification of the economy of Alberta with particular emphasis on water-based sectors in order to present a usable framework for tracing the economic impacts of water development designs.

Several factors influence the accuracy and success of economic forecasts. The scope of this study, however, is limited to the technical factors involved. These factors are examined using various developments in the relevant fields of economic and statistical theory. If the precise nature of observational and elementary errors is known and annual national accounts fit in with an input-output table, then a method can be devised to revise and update a base-year input-output table—and predictions based thereon—using the most recent national accounts data.

Plan of Study

Chapter II outlines the characteristics of scientific predictions

and notes the necessary conditions under which scientific predictions must be generated to facilitate subsequent verification and accuracy analyses of predictions. The purpose of economic predictions is noted and linked with the historical development of economic theory and policy. Chapter III describes the input-output technique of generating intermediate demand predictions and the calculation of prediction errors for verification and accuracy analysis. Methods of adjusting intermediate demand predictions for price and technological changes are discussed in the Appendix to Chapter III. In Chapter IV the statistical structure and properties of input-output prediction errors are presented with major references to works by Henri Theil and C. B. Tilanus. Chapter V utilises the results of Chapter IV to present two methods of revising original input-output intermediate demand projections for greater forecasting accuracy. Chapter VI deals with the uses and applications of input-output analysis to the economy of Alberta and provides a scheme for an inter-sectoral classification of the provincial economy. In this chapter use is made of input-output analytical work already done for the Province of Alberta and of the existing system of regional and national accounts data. Where necessary, modifications are introduced to fit in previous work with the objectives of this study. The Conclusion puts forth some proposals for organising the Alberta economic accounts in order to facilitate continuous and consistent revision and improvement of economic forecasts.

CHAPTER II

SCIENTIFIC PREDICTIONS IN ECONOMIC THEORY AND POLICY

Owing to the probabilistic nature of economic interactions, predictions of economic phenomena will not always be correct. However, predictions generated according to a set of scientific procedures are amenable to ex-post verification and accuracy analysis.

Characteristics of Scientific Predictions

A scientific prediction is an assertion about an outcome. It is normally a statement that asserts the occurrence of, say, an event B if an initial event, A, occurs. The relationship, if A then B, will usually have been established from repeated past observations. Scientific predictions may be of the point prediction or interval prediction type, the former giving single future values and the latter a range of future values for each variable. There may be also conditional or unconditional forecasts. A conditional forecast is a statement concerning an unknown event, which will hold only if some other event occurs. Unconditional forecasts do not have any provisos attached to them. Furthermore there may be single predictions, which are confined to one event only, or multiple predictions, which refer to several, interdependent variables. Various combinations of these classes are possible. Input-output forecasts may be categorised as conditional, multiple, point predictions.

To start with, scientific predictions must be generated according to procedures based on theoretical considerations and on

empirical observations obtained beforehand. The basis used by the forecaster, the kinds of observations used, and the theoretical relationships assumed must all be available for scrutiny by others than the forecaster himself.

Next, scientific predictions must be verifiable: "it must be possible to conclude, after a certain time, in an unambiguous way whether the prediction has turned out to be correct or not, and both possibilities must exist; and it must be possible to verify the procedure by which the prediction itself was derived."[1] Verifiability implies the use of clearly defined concepts and an explicit statement of the time span of forecasts.

However refined the techniques used, predictions can generally be expected to deviate from actual observed values. In the case of scientific predictions then, a probability statement about the relationship between predicted and realised values should be given. For instance, one may have a point prediction of the median type and state that there is an equal probability of the prediction exceeding or falling below the realised value. Alternatively there may be an "unbiased prediction," implying that the prediction has been derived in such a way that the resulting forecasting error has zero mean. For purposes of verification an interval prediction should be accompanied by a probability statement giving the probability that the prediction will be fulfilled. Generally, a trade-off relation exists between precision and reliability of interval predictions; that is, a greater precision (smaller interval) can only be obtained at the cost of less reliability (smaller probability of fulfilment). Multiple predictions

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[1]Henri Theil, Economic Forecasts and Policy (Amsterdam: North-Holland Publishing Co., 1965), p. 11.

are sometimes required to meet the additional test of internal consistency, which is concerned with the question "whether forecasts should obey the same equations and inequalities among themselves as the corresponding actual outcomes do."[1]

Closely related to verification is the process of accuracy analysis of predictions. Accuracy analysis is concerned with forecasting errors or the differences between predictions and outcomes. Specifically accuracy analysis deals with the degree to which the forecasts are imperfect. "A forecast is perfect if it turns out to be identical with the quantity to be predicted. In all other cases it is imperfect—which is usually the case."[2] Depending on the frequency and the seriousness of imperfect forecasts, the forecaster may then reexamine the whole forecasting procedure and decide to retain or revise his procedures. The seriousness of imperfect forecasts can only be judged with reference to the purpose of predictions.

Predictions are normally intended to influence the actual behaviour of individuals or to guide such behaviour towards the attainment of some overall policy goals. Viewed thus, the purpose of economic predictions may be linked with the historical development of economic theory and policy.

Most prominent pre-Marshallian economic writers and a number of them since Marshall's time chose the study of economics because they wished to solve problems of economic policy in a systematic way. The names Smith, Malthus, Ricardo, Mill, Wicksell, and Wicksteed might be mentioned as

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[1] Ibid., p. 14.

[2] Ibid., p. 23.

examples. The following passage from Marshall's book, Industry and Trade, illustrates how early thinkers related theory to prediction and policy.

> Explanation is simply prediction written backwards; and, when fully achieved, it helps towards prediction. A chief purpose of every study of human action should be to suggest the probable outcome of present tendencies; and thus to indicate, tacitly if not expressly, such modifications of those tendencies, as might further the well-being of mankind.[1]

In the development of economic thought the need to predict was brought to the forefront by the rise and fall of the concept of the economic machine.

The Concept of the Economic Machine

Fathered by Adam Smith and his followers, the concept of the economic machine viewed an economic system as a mechanical system that, once properly set in motion, would work regularly and satisfactorily. Obviously, the question of time was extracted from the main writings about the working of the system. However, in addition, the economic machine was said to require no form of human servicing or control, for it was regulated by nature's invisible hands.

As social and economic organisation became more involved and as knowledge of economic conditions received different interpretation, the economic machine was observed to fail in certain parts. It was then thought that some form of regulation would help the machine along. Opinions diverged as to who should undertake regulation and how much regulation to apply. In any case, regulation required a study of the causes and consequences of various economic phenomena and a foreknowledge of the effects of any measures applied to the system.

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[1]Alfred Marshall, Industry and Trade (London, Macmillan, 1919), p. 7.

In the twentieth century fears of economic stagnation and violent economic fluctuations, culminating in the Great Depression, led eminent economists to propose governmental economic controls. The post-war electorates vested their governments with a number of economic obligations. To the extent that policies and controls require predictions, the events outlined so far prompted academic and practising economists and statisticians to develop various tools as aids to forecasting. One of the widely used techniques of economic forecasting is input-output analysis.

CHAPTER III

INPUT-OUTPUT PREDICTIONS AND PREDICTION ERRORS

Input-Output Forecasting Methods

Input-output forecasting methods characteristically ensure that the output of each industry is consistent with intermediate and final demands for the industry's products. Input-output forecasts are sometimes called consistent forecasts for this reason. All consistent forecasts rest on the assumption that economic structural relations do not change significantly over the projection period, or that the relations change according to the allowances made for certain changes anticipated in advance.

An input-output table, showing the way in which the outputs of different sectors of the economy are distributed among various recipients, consists of the following major components[1]:

1. The "interindustry flow" matrix or the "intermediate use" matrix, which is a square matrix of industries producing goods and services. The rows of the matrix comprise the delivering industries, and the columns comprise the receiving industries.
2. The "primary inputs" matrix, designating the use of basic factor services by various sectors.
3. The "final demand" matrix, which records the end-product demand for the outputs of various sectors.

The above matrices may be arranged as shown in the following schematic representation of a static, open, input-output table:

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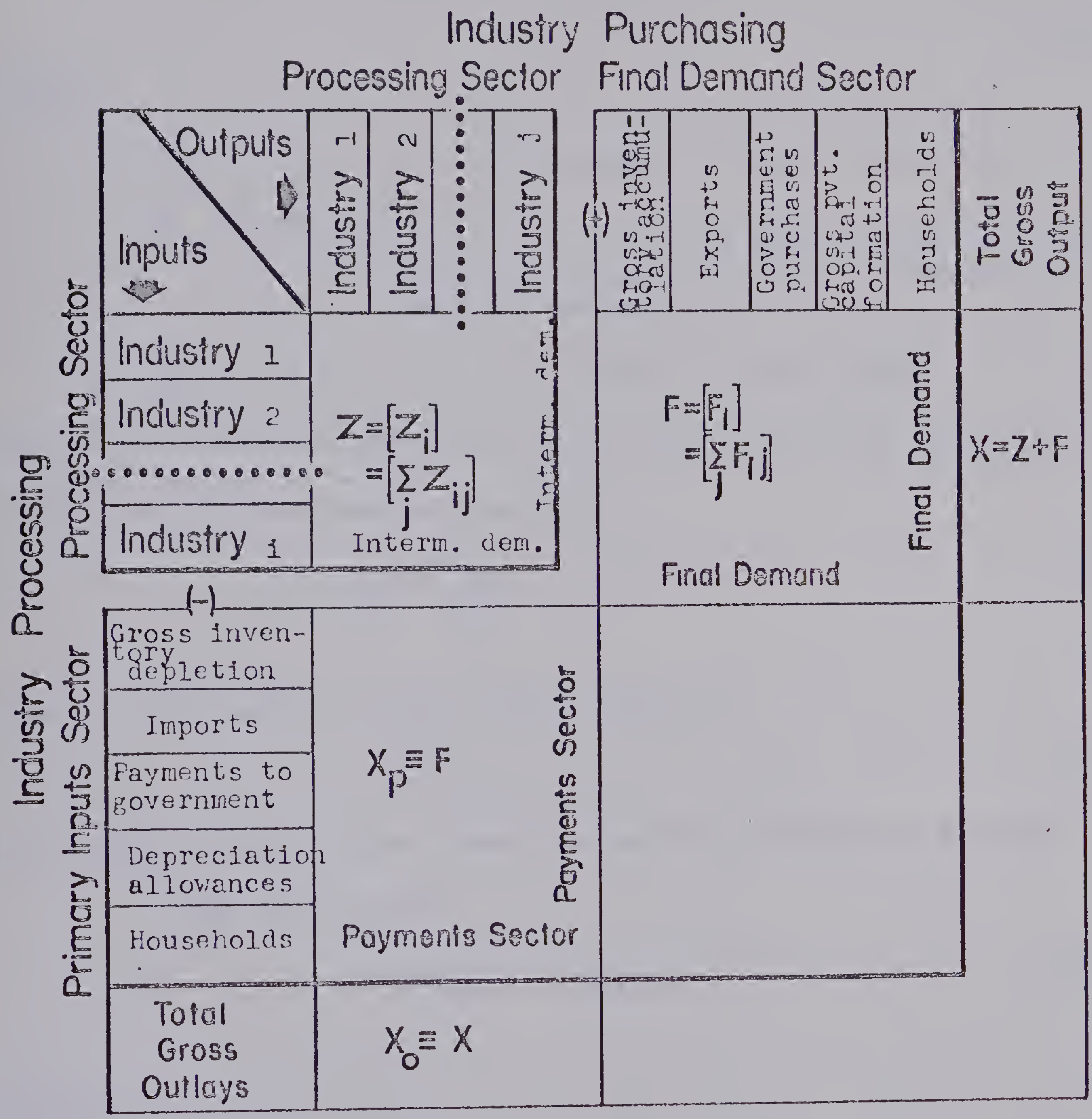
[1]The entries in an input-output table as originally conceived represented physical units. In subsequent uses, however, the entries have been made to represent elements in money terms as a common denominator.

Figure 1

SCHEMATIC REPRESENTATION OF A STATIC,
OPEN INPUT-OUTPUT TRANSACTIONS TABLE



Algebraically the input-output model may be represented as a set of linear equations, the equation for each sector showing the total output of that sector as a sum of sales to other industries, plus the sales to the final demand sector:

$$X = Z + f \qquad (1)$$

where

$X = [X_i]$, i = 1, ..., n, is a vector of total production,

$Z = [Z_i] = [\sum_j Z_{ij}]$ i,j = 1, ..., n, is a square matrix of intermediate deliveries by the i-th industry to the j-th industry.

$f = [f_i]$, i = 1, ..., n, is a vector of final demand.

The (i,j)-th input coefficient is the ratio between the deliveries from i to j and j's total production value,

$$a_{ij} = \frac{Z_{ij}}{x_j}, \text{ i,j} = 1, \ldots, n, \qquad (2)$$

Denoting the matrix of input coefficients by

$$A = [a_{ij}], \text{ i,j,} = 1, \ldots, n, \qquad (3)$$

the system of definitional equations may be developed as follows:

From (2) and (3):

$$Z = [\sum_j Z_{ij}] = [\sum_j a_{ij} x_j] = AX \qquad (4)$$

Using (1):

$$\begin{aligned} Z &= A\,(Z + f) \\ &= (I - A)^{-1} Af \\ &= [(I - A)^{-1} - I]f \qquad , \qquad (5) \end{aligned}$$

The matrix of parameters, $[(I - A)^{-1} - I]$, is the matrix multiplier less the unit matrix. Equation (5) is the basic relation used in input-output

forecasting. Applying the matrix of parameters from a base year t to an exogenously specified vector of final demand of a later year $t + \tau$ $(\tau > o)$, a conditional prediction of intermediate demand for the year $t + \tau$ may be made thus:

$$Z^*_{t\tau} = \text{predicted intermediate demand} = \left[(I - A_t)^{-1} - I\right] f_{t+\tau} \quad , (6)$$

Two major steps are involved in simple input-output forecasting. First, final demand in the year $t + \tau$ is predicted by some ancillary device; second, the base year input-output table is projected on the basis of the predicted changes in final demand. Some usable techniques for making final demand projections are presented in Section A of the Appendix to Chapter III. Price changes and technological changes make it necessary to introduce some modifications in simple forecasting equations. Adjustments for price and technological changes are discussed in Section B and Section C, respectively, of the Appendix to Chapter III.

Consistent scientific forecasts are not necessarily perfect forecasts. Hence the analysis of the input-output prediction errors form the next integral part of the forecasting procedure.

Prediction Errors

In discussing prediction errors, a distinction must be made between ex-ante predictions and ex-post predictions. Ex-ante predictions are forecasts made for a finite future time period. Such predictions must wait until the predicted period comes about and realized values are recorded before prediction errors can be worked out. Ex-post predictions are 'forecasts' that refer to a time period that has already been realized. Such forecasts are of a theoretical interest for judging the validity of a forecasting model. Ex-post predictions can be compared with realized values

at the same time as the predictions are made. One may choose to verify and analyse input-output predictions by computing relative prediction errors or logarithmic prediction errors.

Relative Prediction Errors

A relative prediction error may be defined as

$$e_r = \frac{z^* - z}{z} \quad , \tag{7}$$

where, e_r = relative prediction error,

z^* = predicted value,

z = realized value.

As an indicator of the quality of predictions, the relative prediction error has the disadvantage that, apart from sign, it is not symmetric in z^* and z. If, for instance, $z^* = 1/2\ z$, then e_r is minus 50 percent; if, conversely, $1/2\ z^* = z$, then e_r is plus 100 percent. Symmetry between z^* and z may be obtained by computing logarithmic prediction errors.

Logarithmic Prediction Errors

A logarithmic prediction error may be defined as the natural logarithm of the ratio between prediction and realization:

$$e = \log \frac{z^*}{z} \quad , \tag{8}$$

where, e = logarithmic prediction error.

Logarithmic errors are algebraically smaller than relative errors.[1]

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[1]That e is smaller than e_r may be shown as follows:
Using the relation (7), (8) may be written as:

$$e = \log (1 + e_r).$$

The relative error minus the logarithmic error,

$$\Delta = e_r - \log (1 + e_r) \quad ,$$

has a first derivative $1 - \dfrac{1}{1+e_r}$, which is zero for e_r (and e) equal to zero.

The second derivative, $\dfrac{1}{(1+e_r)^2}$, is positive.

Therefore, the function Δ has a minimum of zero at the point zero and is positive elsewhere.

Hence $\log (1 + e_r) < e_r$; that is, $e < e_r$, except for the trivial case $e_r = 0$.

Ex-post Logarithmic Prediction Errors of Intermediate Demand

It was shown in (6) above that

$$Z^*_{t\tau} = \text{predicted intermediate demand} = \left[(I - A_t)^{-1} - I\right] \delta_{t+\tau}$$

Let

$Z^*_{it\tau}$ = a prediction of the total intermediate demand for the products of sector i, based on the input-output table of year t, and the prediction is τ years ahead.

$Z_{i,t+\tau}$ = observed or realised intermediate demand for the products of sector i in year $t + \tau$.

Then, the ex-post logarithmic prediction error of intermediate demand may be defined as

$$e_{it\tau} = \log \frac{Z^*_{it\tau}}{Z_{i,t+\tau}}$$

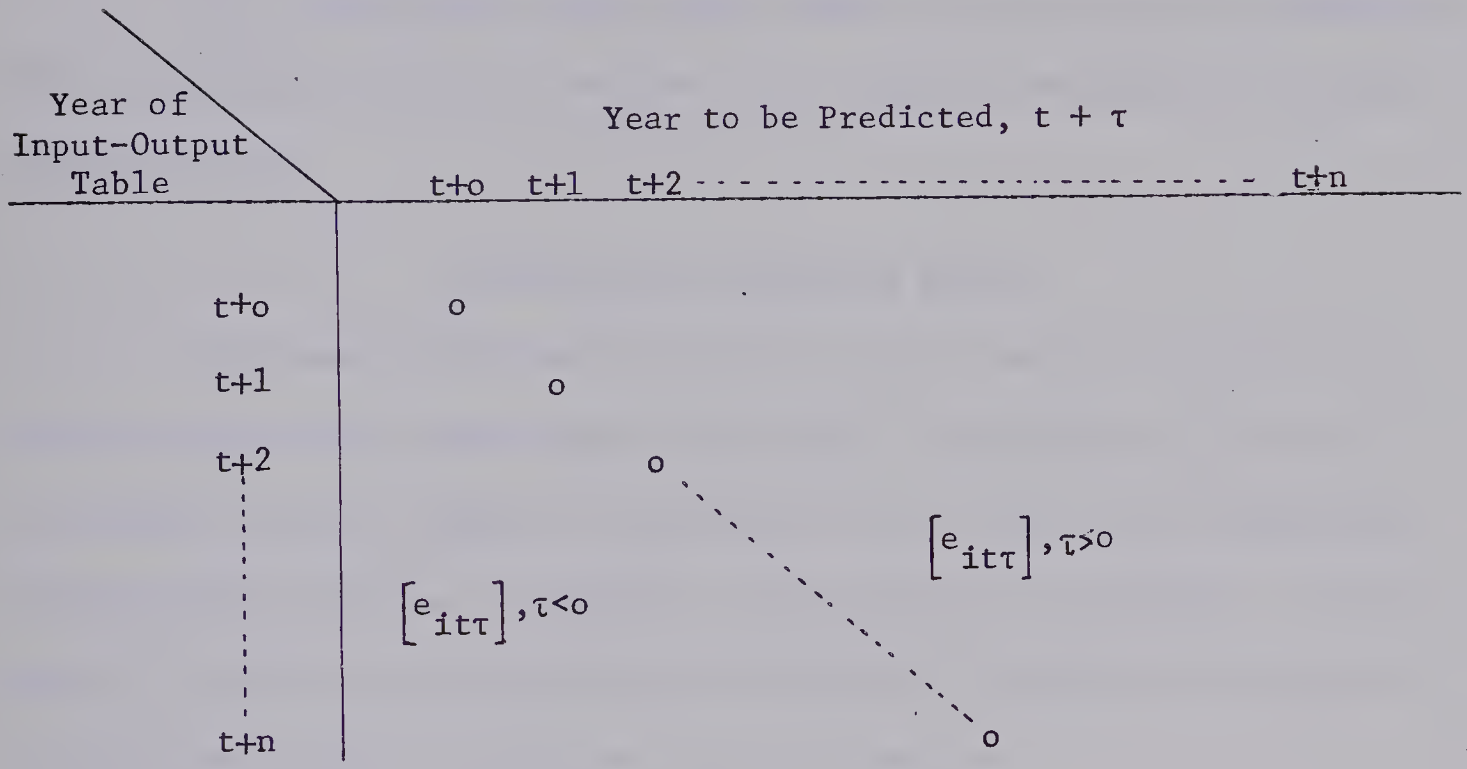
or, (9)

$$e_{it\tau} = \log Z^*_{it\tau} - \log Z_{i,t+\tau}$$

If input-output forecasts are made over a number of years and if tables of several consecutive years are used, then the prediction errors for each sector may be presented in the form of a matrix. The rows of such a matrix would correspond to the years of the input-output table used, and the columns would refer to the years of the intermediate demand value that is forecast. The arrangement of such a matrix is shown below:

Figure 2

MATRIX OF EX-POST PREDICTION ERRORS OF THE INTERMEDIATE DEMAND FOR THE PRODUCTS OF SECTOR i



The diagonal zeros represent the trivially perfect predictions obtained when the input-output table of the prediction year is used in ex-post forecasts. Elements above the zero diagonal refer to "forecast errors," those below the diagonal to "backcast errors." In order to examine the statistical structure and properties of input-output prediction errors, the matrices of prediction errors may be aggregated in various ways over different sectors and time spans.

CHAPTER IV

THE STATISTICAL STRUCTURE OF INPUT-OUTPUT PREDICTION ERRORS

The logarithmic input-output prediction errors may be aggregated over forecasting periods to analyse the statistical structure of the prediction errors.

Mean Square Prediction Errors

The mean square prediction error is an average of squared prediction errors over input-output base years.[1] In squaring the errors it is assumed that the degree of seriousness of an error can be represented by its square; that is, a quadratic loss criterion is applied.[2] For each given i (sector) and $t+\tau$ (time span) combination, averages are taken over t and the mean square prediction errors defined as:

$$m_{i\tau} = \frac{1}{n-\tau} \sum_t e^2_{it\tau} \quad , \tag{1}$$

where,

n = the length of the time span, including the base period and the period of prediction;

$$e_{it\tau} = \log \frac{Z^*_{it\tau}}{Z_{i,t+\tau}} = \log \frac{\text{Prediction}}{\text{Realization}} \quad .$$

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[1] C. B. Tilanus, Input-Output Experiments: The Netherlands 1948 - 1961 (Rotterdam: Rotterdam University Press, 1966), p. 57; G. Rey and C. B. Tilanus, "Input-Output Forecasts for the Netherlands, 1949 - 1958," Econometrica XXXI (1963), 454-463; Henri Theil, Applied Economic Forecasting (Amsterdam: North-Holland Publishing Co., 1966), p. 176.

[2] Harald Cramer, Mathematical Methods of Statistics (Princeton: Princeton University Press, 1946), pp. 213-214, 478-490; Alexander M. Mood, Introduction to the Theory of Statistics (New York: McGraw-Hill, 1950), p. 149; Tjalling C. Koopmans and W. C. Hood, eds., Studies in Econometric Methods (New York: John Wiley and Sons, 1953), p. 130; Leonard J. Savage, The Foundations of Statistics (New York: John Wiley and Sons, 1954), pp. 81-82, 232-234; Carl F. Christ, Econometric Models and Methods (New York: John Wiley and Sons, 1966), pp. 257-262, 267-269.

If $\qquad e_{it\tau} < 1$

then $\qquad m_{it\tau} < e_{it\tau}$, since $m_{i\tau}$ are averages of squares of $e_{it\tau}$.[1]

The square root of $m_{i\tau}$, or the root mean square error, converts the $m_{i\tau}$ back to the dimension of the original errors.

Mean square prediction errors give an indication of the factors which influence the quality of forecasts. These factors are the time span τ over which prediction takes place and the change in the composition of the intermediate demand for the products of sector i. Generally, the errors increase in magnitude as forecasts are made over longer periods. However, the errors increase less than proportionally with the time span. This phenomenon has been theoretically explained by Tilanus and Theil, both of whom make use of the cumulation rule of elementary input-output prediction errors and introduce the role of observational errors in their theoretical explanations.[2]

The Cumulation Rule of Input-Output Prediction Errors[3]

The cumulation rule states that the logarithmic prediction error of the intermediate demand for the products of the i-th sector, based on the table for year t, for the future year $t+\tau$, may be approximated by summing all errors one year ahead (elementary errors) from, and including,

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[1]Tilanus, op. cit., p. 57.

[2]Tilanus, Ibid., pp. 94-123; Theil, op. cit., pp. 176-255.

[3]The cumulation rule is expounded and a theoretical derivation of the rule is presented in Tilanus, Ibid., pp. 94-100; also Theil, Ibid., pp. 208-211, 239-243.

year t up to the year preceding $t+\tau$:

$$e_{it\tau} = \sum_{s=t}^{t+\tau-1} e_{is1} + d_{it\tau} \quad , \qquad (2)$$

where the discrepancy from the cumulation rule, $d_{it\tau}$, is of the second and higher order of smallness. This being so, the discrepancies may be neglected and the cumulation rule written as:

$$e_{it\tau} \approx \sum_{s=t}^{t+\tau-1} e_{is1} \, , \; (\tau > o) \quad , \qquad (3)$$

For time spans of about two to three years the cumulation rule may be expected to be almost exact; for longer time spans small discrepancies may appear. The discrepancies would be small owing to the simultaneous effect of two different causes. One cause is that the technical input coefficients do not change very much from year to year; the other is that the final demand components move approximately proportionally over time.

The Role of Measurement Errors in Input-Output Forecasting[1]

Measurement errors in input-output forecasting can occur when a base-year input-output table is compiled and when the realisation figures of intermediate and final demands in the predicted year are recorded. Hence any observed value of a variable is composed of the "true" value of the variable and the measurement error component. Neither of the components are individually observable. The observed flows of an input-output table may be decomposed as follows:

$$\begin{array}{ll} \text{observed total output} & = X = \bar{X} + \tilde{X} \\ \text{observed intermediate output} & = Z = \bar{Z} + \tilde{Z} \\ \text{observed final demand} & = f = \bar{f} + \tilde{f} \end{array} \quad . \qquad (4)$$

"True" values are indicated by a bar, –; measurement error values are indicated by a tilde, ˜.

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[1]Tilanus, Ibid., pp. 101-105; Theil, Ibid., pp. 211-213.

The matrix of input coefficients is similarly decomposed into

$$A = \bar{A} + \tilde{A} \quad , \tag{5}$$

whose elements may be written as

$$a_{ij} = \bar{a}_{ij} + \tilde{a}_{ij} \; .$$

Further,

$$a_{ij} = \frac{\bar{Z}_{ij} + \tilde{Z}_{ij}}{\bar{X}_j + \tilde{X}_j} = \frac{\bar{Z}_{ij}}{\bar{X}_j} \; \frac{1 + \dfrac{\tilde{Z}_{ij}}{\bar{Z}_{ij}}}{1 + \dfrac{\tilde{X}_j}{\bar{X}_j}} = \bar{a}_{ij} \; \frac{1 + \dfrac{\tilde{Z}_{ij}}{\bar{Z}_{ij}}}{1 + \dfrac{\tilde{X}_j}{\bar{X}_j}} \; . \tag{6}$$

In (6), $\bar{a}_{ij}$ is termed the "true" input coefficient.

$\tilde{Z}_{ij}/\bar{Z}_{ij}$ and $\tilde{X}_j/\bar{X}_j$ are termed the relative measurement errors.

If the relative measurement errors are small, the (i-j)-th element of the error matrix $\tilde{A}$ is approximately equal to the product of the corresponding element of the true input coefficient matrix and the difference between the relative measurement errors as shown below:

$$\begin{aligned}
\tilde{a}_{ij} &= a_{ij} - \bar{a}_{ij} \\
&= \bar{a}_{ij} \left[\frac{1 + \dfrac{\tilde{Z}_{ij}}{\bar{Z}_{ij}}}{1 + \dfrac{\tilde{X}_j}{\bar{X}_j}} - 1 \right]
\end{aligned}$$

$$\begin{aligned}
\therefore \quad \tilde{a}_{ij} &\approx \bar{a}_{ij} \left(1 + \frac{\tilde{Z}_{ij}}{\bar{Z}_{ij}} - \frac{\tilde{X}_j}{\bar{X}_j} - 1\right) \\
&= \bar{a}_{ij} \left(\frac{\tilde{Z}_{ij}}{\bar{Z}_{ij}} - \frac{\tilde{X}_j}{\bar{X}_j}\right) \quad ,
\end{aligned} \tag{7}$$

Input-output forecasts of intermediate demand given final demand may be decomposed as follows:

$$Z^*_{t\tau} = \bar{Z}^*_{t\tau} + \tilde{Z}^*_{t\tau} \quad , \qquad (8)$$

where the "true" forecast is

$$\bar{Z}^*_{t\tau} = \left[(I - \bar{A}_t)^{-1} - I \right] \bar{f}_{t+\tau} \quad . \qquad (9)$$

In order to include the measurement error component of the prediction, $\tilde{Z}^*_{t\tau}$, the matrix multiplier is expanded as follows:

$$(I - A_t)^{-1} = (I - \bar{A}_t - \tilde{A}_t)^{-1} = \left[I - (I - \bar{A}_t)^{-1}\tilde{A}_t \right]^{-1} (I - \bar{A}_t)^{-1}$$

$$\approx (I - \bar{A}_t)^{-1} + (I - \bar{A}_t)^{-1}\tilde{A}_t(I - \bar{A}_t)^{-1} \quad .\text{[1]} \qquad (10)$$

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[1]Expression (10) is derived as follows:

Decomposing the matrix of input coefficients according to expression (5):

$$(I - A_t)^{-1} = (I - \bar{A}_t - \tilde{A}_t)^{-1}$$

Post-multiplying both sides by an identity matrix:

$$(I - A_t)^{-1} I = (I - \bar{A}_t - \tilde{A}_t)^{-1} (I - \bar{A}_t)(I - \bar{A}_t)^{-1}$$

$$\therefore (I - A_t)^{-1} = I^{-1}(I - \bar{A}_t - \tilde{A}_t)^{-1}(I - \bar{A}_t)(I - \bar{A}_t)^{-1}$$

$$= \left[I - (\bar{A}_t - \tilde{A}_t)^{-1} \right]^{-1} (I - \bar{A}_t)(I - \bar{A}_t)^{-1}$$

$$= \left[(I - \bar{A}_t) - (\bar{A}_t - \tilde{A}_t)^{-1}(I - \bar{A}_t) \right]^{-1} (I - \bar{A}_t)^{-1}$$

$$= \left[(I - \bar{A}_t) - (\bar{A}_t^{-1} I - I - \tilde{A}_t^{-1} I + \tilde{A}_t^{-1}\bar{A}_t) \right]^{-1} (I - \bar{A}_t)^{-1}$$

$$= \left[I - (I^{-1}\tilde{A}_t - \bar{A}_t^{-1}\tilde{A}_t) \right]^{-1} (I - \bar{A}_t)^{-1}$$

$$= \left[I - (I - \bar{A}_t)^{-1}\tilde{A}_t \right]^{-1} (I - \bar{A}_t)^{-1}$$

$$\approx (I - \bar{A}_t)^{-1} + (I - \bar{A}_t)^{-1}\tilde{A}_t(I - \bar{A}_t)^{-1}$$

Then for the observed forecast, $Z^*_{t\tau}$:

$$Z^*_{t\tau} = \left[(I - A_t)^{-1} - I\right] f_{t+\tau}$$

$$\approx \left[(I - \bar{A}_t)^{-1} - I + (I - \bar{A}_t)^{-1}\tilde{A}_t(I - \bar{A}_t)^{-1}\right](\bar{f}_{t+\tau} + \tilde{f}_{t+\tau})$$

$$= \bar{Z}^*_{t\tau} + \left[(I - \bar{A}_t)^{-1} - I\right]\tilde{f}_{t+\tau}$$

$$+ (I - \bar{A}_t)^{-1}\tilde{A}_t(I - \bar{A}_t)^{-1}(\bar{f}_{t+\tau} + \tilde{f}_{t+\tau}) \quad , \qquad (11)$$

$$\therefore \tilde{Z}^*_{t\tau} = \left[(I - \bar{A}_t)^{-1} - I\right] \tilde{f}_{t+\tau} + (I - \bar{A}_t)^{-1}\tilde{A}_t(I - \bar{A}_t)^{-1}(\bar{f}_{t+\tau} + \tilde{f}_{t+\tau}) \ .$$

Using $(I - \bar{A}_t)^{-1} - I = (I - \bar{A}_t)^{-1}\bar{A}_t$ and neglecting the second order terms involving products of the elements of $\tilde{A}_t$ and $\tilde{f}_t$,

$$\tilde{Z}^*_{t\tau} \approx (I - \bar{A}_t)^{-1}\left[\bar{A}_t\tilde{f}_{t+\tau} + \tilde{A}_t(I - \bar{A}_t)^{-1}\bar{f}_{t+\tau}\right] \quad , \qquad (12)$$

The relation (12) shows that the measurement error component of the forecast is equal to a linear combination of the measurement errors of the sector input coefficients $(\tilde{A}_t)$ in the base year plus a linear combination of the measurement errors of the final demand vector $(\tilde{f}_{t+\tau})$ in the predicted year.

<u>The Decomposition of Input-Output Forecasting Errors</u>

The logarithmic input-output forecasting errors for sector i may be decomposed into the "true" error and measurement error components as follows:

$$e_{it\tau} = \bar{e}_{it\tau} + \tilde{e}_{it\tau} \quad , \qquad (13)$$

In terms of predicted and realised values, the "true" forecasting error component is

$$\bar{e}_{it\tau} = \log \frac{\bar{Z}^*_{it\tau}}{\bar{Z}_{i,t+\tau}} \quad , \qquad (14)$$

and the measurement error component is

$$\tilde{e}_{it\tau} = e_{it\tau} - \bar{e}_{it\tau} ,$$

or $$\log \frac{\tilde{Z}^*_{it\tau}}{\tilde{Z}_{i,t+\tau}} = \log \frac{Z^*_{it\tau}}{Z_{i,t+\tau}} - \log \frac{\bar{Z}^*_{it\tau}}{\bar{Z}_{i,t+\tau}}$$

$$= \log \frac{Z^*_{it\tau}}{\bar{Z}^*_{it\tau}} - \log \frac{Z_{i,t+\tau}}{\bar{Z}_{i,t+\tau}}$$

$$= \log \frac{\bar{Z}^*_{it\tau} + \tilde{Z}^*_{it\tau}}{\bar{Z}^*_{it\tau}} - \log \frac{\bar{Z}_{i,t+\tau} + \tilde{Z}_{i,t+\tau}}{\bar{Z}_{i,t+\tau}}$$

$$= \log \left[1 + \frac{\tilde{Z}^*_{it\tau}}{\bar{Z}^*_{it\tau}}\right] - \log \left[1 + \frac{\tilde{Z}_{i,t+\tau}}{\bar{Z}_{i,t+\tau}}\right]$$

$$\approx \frac{\tilde{Z}^*_{it\tau}}{\bar{Z}^*_{it\tau}} - \frac{\tilde{Z}_{i,t+\tau}}{\bar{Z}_{i,t+\tau}} . \qquad (15)$$

The measurement error component $\tilde{e}_{it\tau}$ thus consists of two separate terms: one term dealing with the measurement error of the forecast, $\log (Z^*_{it\tau}/\bar{Z}^*_{it\tau})$; the other dealing with the measurement error of the realisation, $\log (Z_{i,t+\tau}/\bar{Z}_{i,t+\tau})$.

For $\tau = o$, that is, for the input-output base year, the following theoretical identity in the "true" variable holds:

$$\bar{Z}_t = \left[(I - \bar{A}_t)^{-1} - I\right]\bar{\delta}_{to} \equiv \bar{Z}^*_{to} , \qquad (16)$$

Similarly for the observed variables

$$Z_t = \left[(I - A_t)^{-1} - I\right]\delta_{to} \equiv Z^*_{to} , \qquad (17)$$

Therefore, for $\tau = o$,

$$\bar{e}_{it\tau} \equiv o$$

$$\text{and } e_{it\tau} \equiv o .$$

Hence $\tilde{e}_{it\tau} \equiv o$, since $\tilde{e}_{it\tau} = e_{it\tau} - \bar{e}_{it\tau}$.

Statistical Properties of Input-Output Prediction Errors

In this study the mean square prediction error is taken as an indicator of the quality of forecasts. This indicator has been defined in such a way that the smaller the mean square prediction error, the better is the forecast. It has been mentioned that the mean square prediction error increases less than proportionately with the prediction time span. If this relationship can be shown to hold theoretically, then a useful method of improving economic forecasts can be devised that would utilise the most recent national accounts data. By using more recent data, the time span of forecasts is reduced and hence the prediction error is reduced.

In this section certain statistical assumptions about the constituent parts of the input-output prediction errors are made as a prerequisite to the explanation of the "less than proportional increase" rule.

The following properties were ascribed to input-output matrix prediction errors in the preceding sections:-

1. the cumulative nature of observed prediction errors:

$$e_{it\tau} = \sum_{s=t}^{t+\tau-1} e_{is1} + d_{it\tau} \quad , \qquad \text{[relation (2) above]} .$$

2. the decomposability of observed errors:

$$e_{it\tau} = \bar{e}_{it\tau} + \tilde{e}_{it\tau} \quad , \qquad \text{[relation (13) above]} .$$

where, $\bar{e}_{it\tau}$ = "true" error; $\tilde{e}_{it\tau}$ = measurement error.

The cumulative rule holds both for the "true" error and the measurement error components of prediction errors:

$$\bar{e}_{it\tau} = \sum_{s=t}^{t+\tau-1} \bar{e}_{is1} + \bar{d}_{it\tau} \quad , \tag{18}$$

$$\tilde{e}_{it\tau} = \sum_{s=t}^{t+\tau-1} \tilde{e}_{is1} + \tilde{d}_{it\tau} \quad , \tag{19}$$

$\bar{d}_{it\tau}$ and $\tilde{d}_{it\tau}$ are close to zero.

Combining (18) and (19):

$$e_{it\tau} = \sum_{s=t}^{t+\tau-1} \bar{e}_{is1} + \bar{d}_{it\tau} + \tilde{e}_{it\tau}, \tag{20}$$

where, according to (15)

$$\tilde{e}_{it\tau} \approx \frac{\tilde{Z}^*_{it\tau}}{\bar{Z}^*_{it\tau}} - \frac{\tilde{Z}_{i,t+\tau}}{\bar{Z}_{i,t+\tau}} \quad .$$

The following assumptions are now introduced, regarding the statistical structure of the variables of (20)[1]:-

1. For each sector i the "true" elementary prediction error $\bar{e}_{is1}$ is a random variable with mean μ_i and variance σ_i^2, independent of s and uncorrelated over time. It is also assumed that $\mu_i = o$ for most sectors.[2]

2. $\bar{d}_{it\tau}$ has an expectation which is negligible; a variance, var $\bar{d}_{it\tau}$, which is also small; and is uncorrelated with $\bar{e}_{is1}$ and $\tilde{e}_{it\tau}$. Var $\bar{d}_{it\tau}$, though small, increases rapidly (more than proportionately) with

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[1]Tilanus, op. cit., pp. 107-112; Theil, op. cit., pp. 181-184.

[2]Tilanus, Ibid., p. 108, and Theil, Ibid., pp. 216-217, propose a method for statistically testing the assumption that $\mu_i = o$ for most sectors. Also to show that $\bar{e}_{is1}$ are uncorrelated over time, Theil, Ibid., p. 218, proposes a test by means of an inspection of the signs of the corresponding observed elementary errors, e_{is1}.

the time span.

3. $\tilde{e}_{it\tau}$ has an expectation which is negligible and a variance, var $\tilde{e}_{it\tau}$, which is independent of the time span and the base year. It is also uncorrelated with $\bar{e}_{isl}$.

Using the relation (15), the variance of $\tilde{e}_{it\tau}$ can be written as the sum of the two variances less a double covariance:

$$\text{var}\ \tilde{e}_{it\tau} = \text{var}\left[\log \frac{Z^*_{it\tau}}{\bar{Z}^*_{it\tau}}\right] + \text{var}\left[\log \frac{Z_{i,t+\tau}}{\bar{Z}_{i,t+\tau}}\right] - 2\ \text{cov}\left[\log \frac{Z^*_{it\tau}}{\bar{Z}^*_{it\tau}}\ ,\ \log \frac{Z_{i,t+\tau}}{\bar{Z}_{i,t+\tau}}\right]$$

or, still using (15),

$$\text{var}\ \tilde{e}_{it\tau} \approx \text{var}\left[\frac{\tilde{Z}^*_{it\tau}}{\bar{Z}^*_{it\tau}}\right] + \text{var}\left[\frac{\tilde{Z}_{i,t+\tau}}{\bar{Z}_{i,t+\tau}}\right] - 2\ \text{cov}\left[\frac{\tilde{Z}^*_{it\tau}}{\bar{Z}^*_{it\tau}}\ ,\ \frac{\tilde{Z}_{i,t+\tau}}{\bar{Z}_{i,t+\tau}}\right] \quad , \quad (21)$$

The first right-hand variance, which refers to the relative measurement error of the forecast, does not vary systematically with τ, and therefore is assumed independent of τ. The second right-hand variance is similarly assumed independent of τ.[1] The covariance term may not vanish owing to the possible statistical relationship between the vector $\tilde{z}_{t+\tau}$ and $\tilde{\delta}_{t+\tau}$, which figures in the measurement error of $Z^*_{it\tau}$. However, this covariance does not vary systematically with τ. Hence the variance of $\tilde{e}_{it\tau}$ is assumed independent of τ. If there is no dependence on t either, then var $\tilde{e}_{it\tau}$ becomes a function of the sector index i only.[2]

Given the above statistical assumptions, the following explanation of the "less than proportional increase" rule may be formulated.

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[1]Theil, Ibid., p. 214.

[2]Theil, Ibid., p. 220.

An Explanation of the Less than Proportional Increase of Mean Square Prediction Errors with the Time Span[1]

Using (18) and statistical assumption (1), the expected value of the true prediction error τ years ahead is:

$$E\left[\bar{e}_{it\tau}\right] = \tau\mu_i \quad , \tag{22}$$

where the expectation of the discrepancy $\bar{d}_{it\tau}$ is neglected because the discrepancies are very small. The variance of $\bar{e}_{it\tau}$ is

$$\text{var}\left[\bar{e}_{it\tau}\right] = \tau\sigma_i^2 + \text{var}\,\bar{d}_{it\tau} \quad . \tag{23}$$

The mean square is obtained by adding the squared mean to the variance[2]

$$E\left[\bar{e}^2_{it\tau}\right] = \tau\sigma_i^2 + \text{var}\,\bar{d}_{it\tau} + \tau^2\mu_i^2 \quad , \tag{24}$$

Adding $\tilde{e}_{it\tau}$ to $\bar{e}_{it\tau}$ we obtain $e_{it\tau}$, as in (20):

$$e_{it\tau} = \tilde{e}_{it\tau} + \sum_{s=t}^{t+\tau-1} \bar{e}_{is1} + \bar{d}_{it\tau}$$

By statistical assumption (3) above, which states that the measurement errors comprising $\tilde{e}_{it\tau}$ are random variables with zero mean, the expected value of the observed $e_{it\tau}$ is the same as that of $\bar{e}_{it\tau}$:

$$E\left[e_{it\tau}\right] = \tau\mu_i \quad , \tag{25}$$

Assuming that the measurement errors are uncorrelated with the true errors, the following variance is obtained:

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[1]Tilanus, op. cit., pp. 107-112.

[2]Alexander M. Mood and F.A. Graybill, Introduction to the Theory of Statistics (New York: McGraw-Hill, 1965), pp. 166-167, 176-177; D. Blackwell, "Conditional Expectation and Unbiased Sequential Estimation," Annals of Mathematical Statistics, XVIII (1947), 105-110.

$$\text{var}\left[e_{it\tau}\right] = \text{var}\ \tilde{e}_{it\tau} + \tau\sigma_i^2 + \text{var}\ \bar{d}_{it\tau} \quad , \qquad (26)$$

and the mean square:

$$E\left[e_{it\tau}^2\right] = \tau\sigma_i^2 + (\tau^2\mu_i^2 + \text{var}\ \bar{d}_{it\tau}) + \text{var}\ \tilde{e}_{it\tau} \ , \qquad (27)$$

where $\mu_i = o$ is understood to hold for most sectors.

The relation (27) represents the explanation sought in this section. The first right-hand term of (27) increases proportionately with τ, the time span. The terms between parenthesis, $(\tau^2\mu_i^2 + \text{var}\ \bar{d}_{it\tau})$, increase more than proportionately with τ, but they have small values. The measurement error term, (var $\tilde{e}_{it\tau}$), is a constant. The observed discrepancies from the cumulation rule being small, it may be assumed that the discrepancy term is small compared with the measurement error term. Moreover, considering that $\mu_i = o$ for most sectors, it may be concluded that the functional relationship between the expected squared prediction error, $E\left[e_{it\tau}^2\right]$, and the time span, τ, is approximately a straight line with a positive intercept (var $\tilde{e}_{it\tau}$) and a positive, but less than unity, slope (σ_i^2).[1] Hence mean square prediction error increases less than proportionately with an increase in the time span of prediction.

The "less than proportional increase" rule implies that long-term input-output forecasts may be made more accurate by a continual revision of original forecasts over the prediction period. Greater accuracy is obtained by reducing the time span of input-output forecasts. The time span of forecasts is reduced by using the most recent inter-industry and other data available from national income accounts. The next step then is to see how the national accounts data of, say, year $t + q$, $(o < q < \tau)$, may be used to revise the input-output table—and forecasts—of year t when predicting year $t + \tau$.

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[1]In practice, the straight line may be regarded as an approximation of an exponential relationship.

CHAPTER V

CORRECTING INPUT-OUTPUT PREDICTIONS USING MORE RECENT NATIONAL ACCOUNTS DATA

The integration of input-output predictions and national accounts data is conceptually possible because input-output models and standard national accounts are closely linked in social accounting systems. A social accounting system is a closed, accounting matrix, which reduces innumerable transactions concerned with production, consumption, accumulation and foreign trade to some kind of order and provides a framework to contain the estimates for any particular economy. It is necessary to consolidate the social accounts into a number of convenient classes to reduce the number of flows in an economic system to manageable proportions. Consolidation is achieved by:

(i) partitioning the matrix into a number of submatrices;

(ii) adding up the matrix within each submatrix;

(iii) discarding intra-class transactions represented by the diagonal elements.

National accounts are formed by partitioning the social accounting matrix into four classes of accounts, viz., production, consumption, accumulation, and the rest of the world. Each of these classes represents a large aggregate. Thus in a national accounting system production is treated as a whole so far as the accounts are concerned, although it may be classified in various ways in supporting tables.

In order to analyse the productive system in detail, the national account for production must be broken down into a number of industries.

When the break-down is achieved, the result is an input-output table. In constructing an input-output table, all non-production accounts may be consolidated into one if a whole picture of the economy is to be retained. The resulting set of input-output accounts then appears as a partitioned matrix consisting of:

(i) a square matrix of intermediate inputs and outputs;

(ii) a column vector of final demand;

(iii) a row vector of primary inputs;

(iv) a single zero which appears at the intersection of (ii) and (iii) and indicates that the non-production accounts are consolidated.

Suppose now that an input-output table for year t is constructed. The table is used to make input-output intermediate demand predictions for a series of future years $t + 1, \ldots\ldots, t + \tau$. Over the years national accounts data, giving a breakdown into industries, becomes available for years $t + 1, \ldots\ldots, t + q$ $(q < \tau)$. Hence the intermediate demand prediction errors, e_{itv}, $v = 1, \ldots\ldots, q$, may be computed. The question posed now is whether the forecasting experience of years $t + 1$ to $t + q$ can be used to improve the predictions of intermediate demand for a future year $t + \tau$.

The Proposed Solution[1]

Add to the logarithm of the prediction $Z^*_{it\tau}$ the following linear combination of the observed prediction errors

$$\sum_{v=1}^{q} \lambda_v e_{itv} , \lambda_v = \text{a series of weights} , \quad (1)$$

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[1]C. B. Tilanus, Input-Output Experiments: The Netherlands 1948 - 1961 (Rotterdam: Rotterdam University Press, 1966), p. 112.

to obtain

$$\log Z^*_{it\tau} + \sum_{v=1}^{q} \lambda_v e_{itv} \quad . \tag{2}$$

Substituting $e_{itv} = \log \dfrac{Z^*_{itv}}{Z_{i,t+v}}$ in (2) gives

$$\log Z^*_{it\tau} + \lambda \sum_{v=1}^{q} \log \left[\frac{Z^*_{itv}}{Z_{i,t+v}} \right] \quad , \tag{3}$$

or,

$$Z^*_{it\tau} \prod_{v=1}^{q} \left[\frac{Z^*_{itv}}{Z_{i,t+v}} \right]^{\lambda_v} \quad . \tag{4}$$

Relation (4) is the proposed general revised forecast of the intermediate demand for the products of sector i in year $t + \tau$. The logarithmic error of the forecast (4) would be

$$e_{it\tau} + \sum_{v=1}^{q} \lambda_v \, e_{itv} \quad . \tag{5}$$

Postulate a special case where $\lambda_1 = \cdots\cdots \lambda_{q-1} = o$, $\lambda_q = -1$. The special case means that one subtracts the most recent observed logarithmic prediction error from the logarithm of the original prediction for year $t + \tau$. The revised prediction for year $t + \tau$ would then be

$$Z^*_{it\tau} \left[\frac{Z^*_{itq}}{Z_{i,t+q}} \right]^{-1} \quad , \tag{6}$$

and the prediction error would be

$$e_{it\tau} - e_{itq} \quad . \tag{7}$$

From the cumulation rule [(3), Chapter IV]:

$$e_{it\tau} - e_{itq} \approx \sum_{s=t+q}^{t+\tau-1} e_{is1} \quad , \qquad (8)$$

Hence the new prediction error is no longer the sum of τ errors in predicting one year ahead, but of only $\tau-q$ such errors. The procedure (6) of revising intermediate demand predictions is termed the statistical correction method.[1]

The Statistical Correction Method

Given the national accounts data for the latest year (t+q), multiply the input-output prediction for year $t+\tau$ ($\tau>q$) based on the input-output table for year t by the ratio between observed realisation in year (t+q) and the prediction for that year:

$$Z^{*(SCM,q)}_{it\tau} = Z^{*}_{it\tau} \frac{Z_{i,t+q}}{Z^{*}_{itq}} \quad , \qquad (9)$$

If, for example, the forecast Z^{*}_{itq} exceeds the corresponding realisation by 5 percent, the statistical correction method amounts to dividing the forecast $Z^{*}_{it\tau}$ by 1.05.

The revised input-output matrix of predicted intermediate demand is, by the statistical correction method,

$$Z^{*(SCM,q)}_{t\tau} = \hat{c}_{tq}\left[(I - A_t)^{-} - I\right]f_{t+\tau} \quad , \qquad (10)$$

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[1]For an analysis of the logarithmic prediction error of the revised forecast and a comparison of the revised method of prediction with the unrevised method see Tilanus, Ibid., pp. 113-115; Henri Theil, Applied Economic Forecasting (Amsterdam: North-Holland Publishing Co., 1966), pp. 223-227.

where $\hat{c}_{tq}$ is a diagonal matrix of correction factors whose i-th diagonal element is $c_{itq} = \frac{Z_{i,t+q}}{Z^*_{itq}}$.

The RAS Method of Revising Input-Output Predictions

The RAS method, developed by Stone and Brown,[1] produces results similar to those produced by the statistical correction method. The RAS method aims at replacing the original matrix of input coefficients, A_t , with a revised matrix which takes the data of year t+q into account. These recent data provide a knowledge of

$$X_{t+q} , \delta_{t+q} , Z_{t+q} = X_{t+q} - \delta_{t+q} .$$

A_t and $\delta_{t+\tau}$ are supposed to be known.

When national accounts data for year t+q became available, the matrix A_t may be checked against these data by comparing the observed Z_{t+q} with the forecast Z^*_{tq} . The observed differences between realisation and prediction figures will indicate the magnitudes and signs of changes in the absorption of the output of each sector i (each row) by all other sectors. It is assumed that the elements of each row of A_t between the years t and t+q are affected by the same multiplicative factor r_{itq} . Each $r_i < 1$ if the demand for the products of the i-th sector decreases over the period t to t+q; $r_i > 1$ if demand increases. Thus the first step is to replace A_t by RA_t such that

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[1]Richard Stone and J.A.C. Brown, "A Long-Term Growth Model for the British Economy," Europe's Future in Figures (Amsterdam: North-Holland Publishing Co., 1962). Also W. E. Deming, Statistical Adjustment of Data (New York: Wiley, 1943), pp. 115-117; R. Stone, "An Econometric Model of Growth: the British Economy in Ten Years Time," Discovery XXII (1961) 216-219; Stone, "A Demonstration Model for Economic Growth," The Manchester School, XXX (1962), 1-14.

$$RA_t X_{t+q} = Z_{t+q} \quad , \qquad (11)$$

where X is the known matrix of total output

R is a diagonal matrix of correction factors whose i-th diagonal element is r_i .

The matrix R of correction factors applies to the rows of the coefficient matrix A_t . The elements of each column of A_t are also affected by a column factor S_{jtq} , which represents changes in the input pattern of each sector. Each S_{jtq} is derived as follows:

From the data for year t+q the following is known:

$$\sum_i a_{ij} x_j = \sum_i Z_{ij} = \text{total intermediate inputs},$$

and

$$\sum_j a_{ij} = \frac{\sum_i Z_{ij}}{x_j}$$

Thus

$$\iota' A_{t+q} = c'_{t+q} \quad , \qquad (12)$$

where, c_{t+q} = known vector of the ratios of total intermediate inputs to total input in year t+q,

ι = a unit vector.

To satisfy (12) each element, a_{ij} of A_t is multiplied by an appropriate S_j, since generally the coefficients matrix A_t may not satisfy the constraint (12). Therefore,

$$\iota' A_t S = c'_{t+q} \quad , \qquad (13)$$

where S is a diagonal matrix whose j-th diagonal element is S_j.

Since both the correction factors R and S operate simultaneously, A_t may be replaced by RA_tS . The diagonals of R and S are such that

$$RA_tSX_{t+q} = Z_{t+q}$$

$$\iota' RA_tS = c'_{t+q} \quad , \qquad (14)$$

The modified forecast for intermediate demand in year $t+\tau$ is then

$$Z^{*(RAS,q)}_{t\tau} = \left[(I - RA_t S)^{-1} - I\right] f_{t+\tau} \quad , \tag{15}$$

In practice, the elements of R and S are derived from iterative computational procedures, which combine mathematical operations with extraneous knowledge about the various factors that bring about changes in input coefficients.

A brief algebraic comparison of the statistical correction method and the RAS method and a discussion of the relative computational ease of the SCM may be found in Tilanus.[1] The RAS method has been worked in the United Kingdom for British and some European data. A discussion of the RAS method, its properties, and the statistical sources and methods used for various test runs may be found in a publication by Bates and Bacharach.[2]

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[1]Tilanus, Ibid., pp. 119-123.

[2]John Bates and Michael Bacharach, "Input-Output Relationships 1954 - 1966," A Programme for Growth, Series 3, edited by Richard Stone (London: Chapman and Hall, 1963).

CHAPTER VI

INPUT-OUTPUT MATRIX PREDICTIONS: ITS USES AND APPLICATIONS TO THE ECONOMY OF ALBERTA

This chapter discusses an empirical input-output framework for the economy of Alberta in a way that utilises the material and results obtained in the previous chapters. The chapter specifies ways in which an Alberta input-output table—and predictions obtained therefrom—would be of use to economic policy makers. Some further theoretical and conceptual issues that would arise in forecasting economic activities by input-output analysis are discussed and an inter-industry classification of the economy of Alberta is presented. Finally, different parts of the study are linked up and described with the aid of a flow diagram in order to indicate the nature of data requirements and information flows in an organised system of input-output prediction analysis.

A Theoretical Explanation and Empirical Description of the System of Inter-industry Relationships in Alberta

An input-output model of Alberta would be set up in terms of commodities and activities, corresponding to the mathematical concepts of variables and functions, and the method of functional separability. Specifically one would have a number of functionally interdependent variables. One would then attempt to group these variables so that they should form functions within the overall function. This breaking up of the overall function into component functions is the mathematical equivalent to the division of the provincial economy into sectors of industry. The accounts of the industrial sectors and other

financial accounts would then be presented in an input-output transactions table.

The input-output transactions table (Figure 1, Chapter III) may be examined from three different angles: a row by row inspection shows how the total output of each sector is allocated among all other sectors. A column by column inspection shows the distribution, by industry of origin, of all the inputs absorbed in the system, thereby emphasising the cost aspect and the internal structure of each individual sector. Finally, a comparison of each column with the corresponding row reveals the industrial balance among all sectors of the provincial economy in terms of the outputs of, and inputs for, each category of goods and services.

The technical coefficients matrix [(2), (3), Chapter III], obtainable from the transactions matrix, will show how much of each kind of input is required per unit of the finished product. Further, the inverse matrix, [$(I - A)^{-1}$], will give the numerical measure of the degree of interdependence between various sectors. An element of the inverse matrix represents the direct and indirect effect on output of, say, industry i of a dollar change, in either direction, in deliveries to final demand from, say, industry j [(5), Chapter III].

Thus, starting from the basic input-output format, one could evaluate the quantitative implications of alternative policies regarding the allocation of primary resources, or various patterns of public works, or industrial and governmental sales and purchases. By the use of input-output forecasting method, given programs of final output in the future may be translated into levels of total and intermediate outputs, and then the impact on manpower, natural resources, and industrial facilities may be traced [(16), Chapter III]. Such an exercise may reveal potential future

bottlenecks in the supply of some key primary and intermediate commodities. The policy problem would then be to maximise production by allocating various scarce factors in an efficient manner. The efficient pattern of allocation may be obtained with the aid of input-output analysis discussed in Appendix A to this chapter.

If a series of input-output tables or tables of prediction errors are available, policy makers may observe, in quantitative terms, the direction and magnitude of changes in inter-industry relations over the years.[1] Furthermore, tables of input-output prediction errors may be integrated with other types of economic statistics to revise economic forecasts for greater accuracy. [Refer relation (9), the SCM, Chapter V.]

One remarkable feature of input-output analysis is that once a basic format has been empirically obtained, additional information may be incorporated in the empirical model to yield more than proportional returns in the form of knowledge about the actual working of an economic system or a part of the system. This feature would make it possible to derive dynamic input-output results for the Province of Alberta. The empirical construction of a dynamic input-output model is discussed in Appendix B to this chapter. As pointed out in the Appendix, the construction of a dynamic input-output model would obviously add to the aggregation problem[2] already present in

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[1]See discussion preceding relation (11), Chapter V, under the heading, "The RAS Method of Revising Input-Output Predictions,"

[2]The problem of aggregation arises in passing from a theoretical model to its statistical analogue owing to two principal factors: first, while in a theoretical model commodities and activities are defined in an abstract manner, in practice a decision has to be made whether an item is a commodity or not and whether a process is an activity or not. Second, while theory may be able to handle an infinite number of commodities and activities in an abstract manner, the number of commodities and activities in practice has to be limited for reasons of consistency and comparability of available data and for computational reasons.

any static industrial classificatory system. Any practical solution to the aggregation problem depends on the purpose for which an input-output table is set up. One of the objectives of this study was to propose an inter-sectoral classification of the economy of Alberta, with emphasis on the water-based sectors. In view of the stated objective the next section presents various aggregate sectors of the economy of Alberta.

Water-based Industrial Sectors: The Province of Alberta

The classification proposed in this section is based primarily on the International Standard Industrial Classification System[1]; on an annual statistical publication of the Provincial Government[2]; and on two studies conducted for the Province.[3] The Wright study provides an input-output format for the Province of Alberta. The Schultz study provides preliminary estimates of various water-use coefficients for selected industries in Edmonton.[4] The following observations may be made as regards the two studies mentioned above and the classification proposed in this study.

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[1]United Nations, Statistical Office, "International Standard Industrial Classification of All Economic Activities (I.S.I.C.)," Statistical Papers, Series M, No. 4 (New York: Statistical Office, United Nations, 1958).

[2]Alberta, Department of Industry and Tourism, Alberta Bureau of Statistics, Annual Review of Business Conditions (Edmonton: Department of Industry and Tourism, Annual).

[3]R. W. Wright, The Alberta Economy: An Input-Output Analysis (Calgary: University of Calgary, 1966); W. M. Schultz, Water Use by Economic Sector: Some Preliminary Estimates (Edmonton: University of Alberta, 1968).

[4]Conclusions about water use in the city of Edmonton may be assumed to hold, with some modifications, in the case of the Province of Alberta.

1. The Wright study aggregated electricity and water into one sector. This one sector will have to be disaggregated.

2. The Schultz study pointed out that the Services sector was one of the principal water-using industrial groups. Therefore, it would seem important to have a more detailed breakdown of the Services sector.

3. The Food and Beverages Manufacturing was found to be another major industrial water user. Here again a further breakdown of foods, beverages, and related industries might be useful.

4. The Wright study classified residential and industrial construction into separate sectors. These may be aggregated into one sector.

5. The gross sales of some other sectors from Wright's study may be aggregated as follows:

Disaggregated Sectors	Aggregate Sector
Primary Metal Metal Fabricating	Primary Metal and Metal Fabricating
Machinery Transportation Equipment Auto Accessories Electrical Products	Machinery and Transportation Equipment
Leather - Textiles Clothing	Leather, Textiles and Clothing
Transport - rail Transport - other Communication	Transport, Communication and Storage

The proposed classification of the Alberta accounts for the entire input-output transactions matrix is contained in Appendix C to this chapter. In the final section different parts of the study are linked up and described in the following flow diagram and the accompanying diagrams of computing sequences for making input-output projections.

The Nature of Data Requirements and Data Handling for Input-Output Forecasting in Alberta

Figure 3 is a schematic representation of the present study.

Stage I: The Modelling Stage

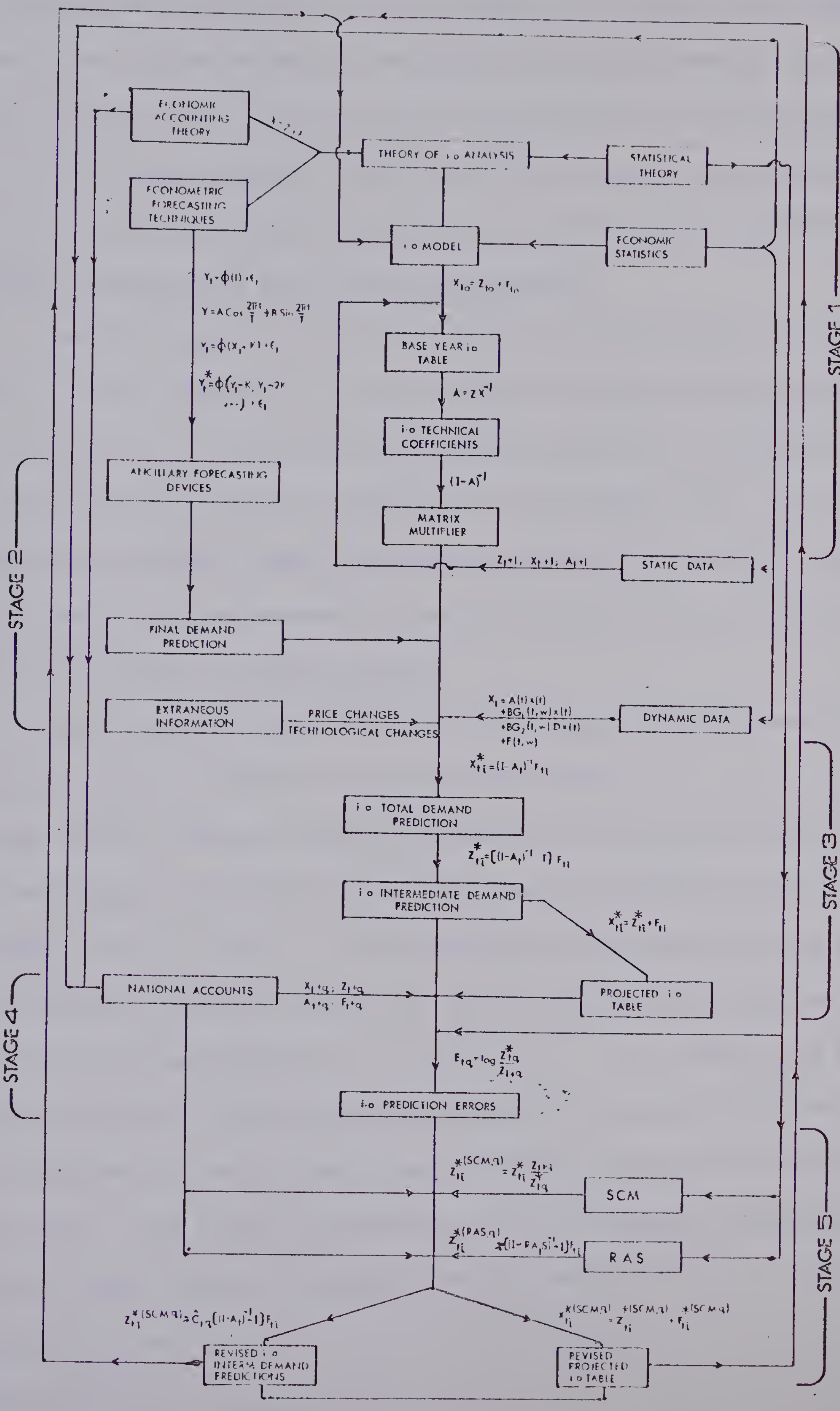
The theory of input-output analysis in general, and of input-output prediction analysis in particular, is derived from economic accounting theory, statistical theory, and econometric forecasting techniques. A combination of these theories with organised statistics on production, distribution, exchange, and consumption enables an empirical input-output model to be set up. From this model the matrix of transactions between various productive sectors, the base-year input-output table [(1) Chapter III] is singled out for the computation of technical coefficients of production [(2), (3), Chapter III] and the matrix multiplier, which is the principal tool of input-output predictions [(5), Chapter III]. The matrix multiplier shows the total—direct and indirect—increased output required from all classified industrial sectors to obtain an additional dollar of final demand from any one of the sectors.

Stage II: The Stage of Pooling Prediction Tools

First, any revised set of crucial economic data that becomes available since the operation of Stage I may be re-run through Stage I to refine the matrix multiplier. Meanwhile, it may be recalled that input-output forecasts are conditional forecasts. The forecasts are conditional upon given values of final demands in the year or years being predicted. Thus second, while the matrix multiplier is being calculated or recalculated, final demand in a series of forthcoming years may be prepared through ancillary econometric forecasting devices. Any one of these devices would typically require sectorally-distributed primary or secondary

Figure 3

A FLOW DIAGRAM OF THE INPUT-OUTPUT MATRIX PREDICTION SYSTEM



statistical time series on household and government expenditures on consumption, private and public investment outlays, and export figures. Third, extraneous information on price changes and technological changes foreseen by an opinion survey of individual industrial experts may be gathered. Fourth, any available data on the possible outcome of current wage and salary negotiations, the course of future wage trends, export markets, the rates of change of population and output may be gathered.

Stage III: The Stage of Input-Output Projections

In this stage the matrix multiplier and the series of final demand forecasts, together with any available extraneous and/or dynamic data are fed into the input-output forecasting equations to obtain the total and intermediate demand predictions [(6), Chapter III]. The total, intermediate, and final demand predictions may then be compiled in an input-output tabular arrangement to yield a projected input-output table.

Stage IV: The Stage of Error Analysis

At the end of Stage III the forecasting system will have yielded a series of economic forecasts τ time periods ahead ($\tau = o,1,2,......$) on the basis of the technical coefficient matrix of the initial time period, t. At the beginning of Stage IV the analysts may be considered to be in the time-period $t + q$ ($o < q < \tau$). The analysts in this stage are availed of observed national accounts figures on total, intermediate, and final demands for the time periods $t + 1$, $t + 2$, ..., $t + q$, which they had previously predicted. So forecasting errors for various productive sectors may be computed to obtain input-output prediction errors [(9) and Figure 2, Chapter III]. This series of prediction errors represents forecasting experience gained before the entire forecast period ($t + \tau$, $\tau = o,1,2,$ ...., $q,...$) is out.[1]

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[1]See supra, Chapter V, p.

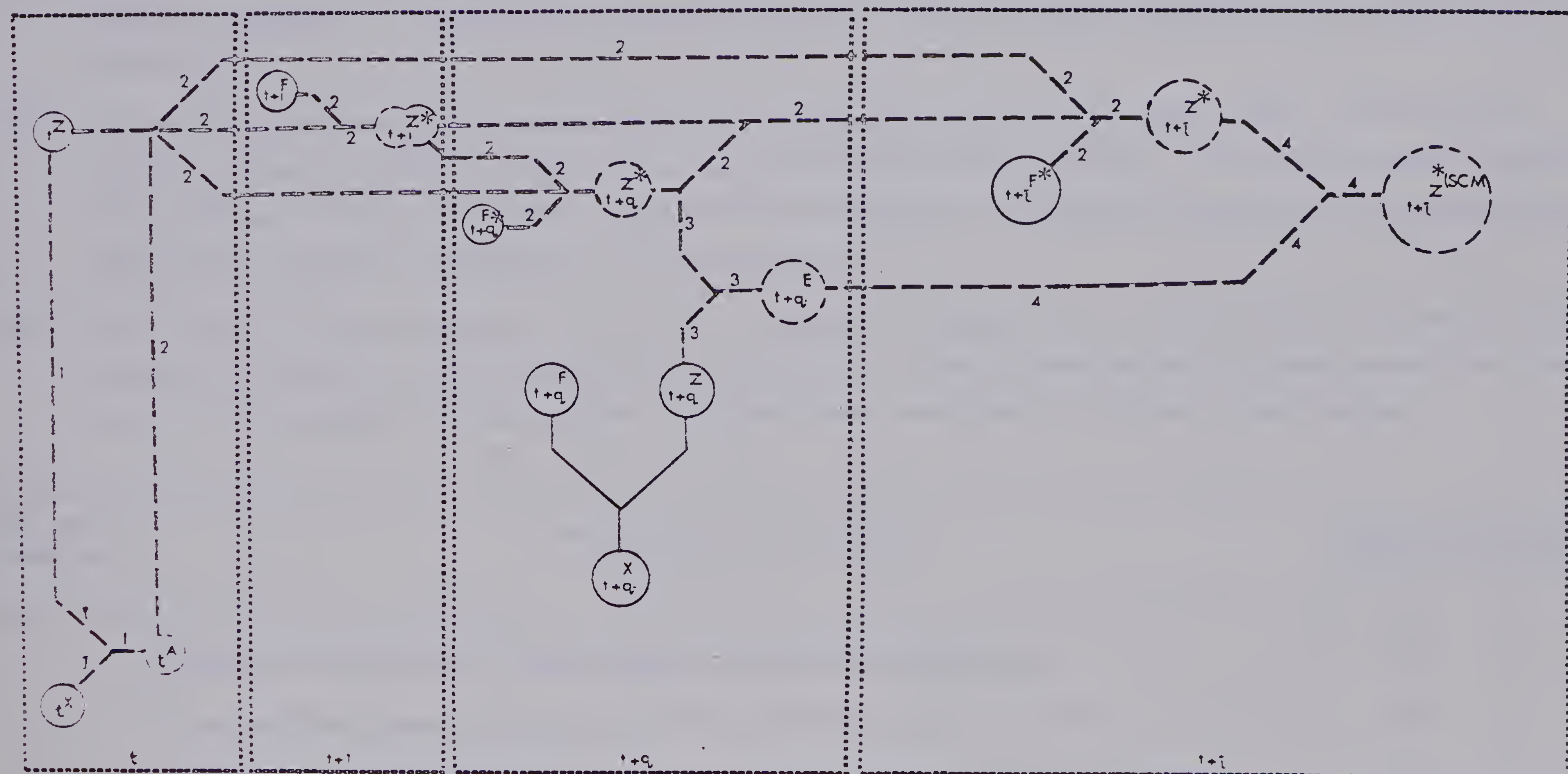
Stage V: The Stage of Corrections

In this stage the forecasting experience of Stage IV is utilised in one of the two alternative correction formulae—the S.C.M. [(9) Chapter V] and the RAS [(15), Chapter V]—presented in the study. The end product is a revised set of predictions for the remaining length of the forecast period ($t + q + 1$, $t + q + 2$, ..., $t + \tau$) and a revised projected input-output table. The freshly projected input-output-table may then be used to initiate a new forecasting period beyond the time period $t + \tau$, and the whole process moves back to Stage I of the forecasting system.

For a satisfactory operation of the forecasting system described above, forecasters need to be furnished with pertinent economic and technological data. The forecasters would utilise the data to generate scientific predictions, which, in turn, would be used as a basis for formulating economic policies. With regard to Alberta, this study has an important implication for the organisation and presentation of economic statistics by official statisticians.

Figure 4

A DIAGRAM OF THE COMPUTING SEQUENCE FOR THE STATISTICAL CORRECTION METHOD



Notes to Figure 4 Diagram

(i) X = total output; Z = intermediate demand matrix; A = technical coefficients matrix; f = vector of final demands; e = matrix of prediction errors. The subscripts preceding each symbol denote the year.

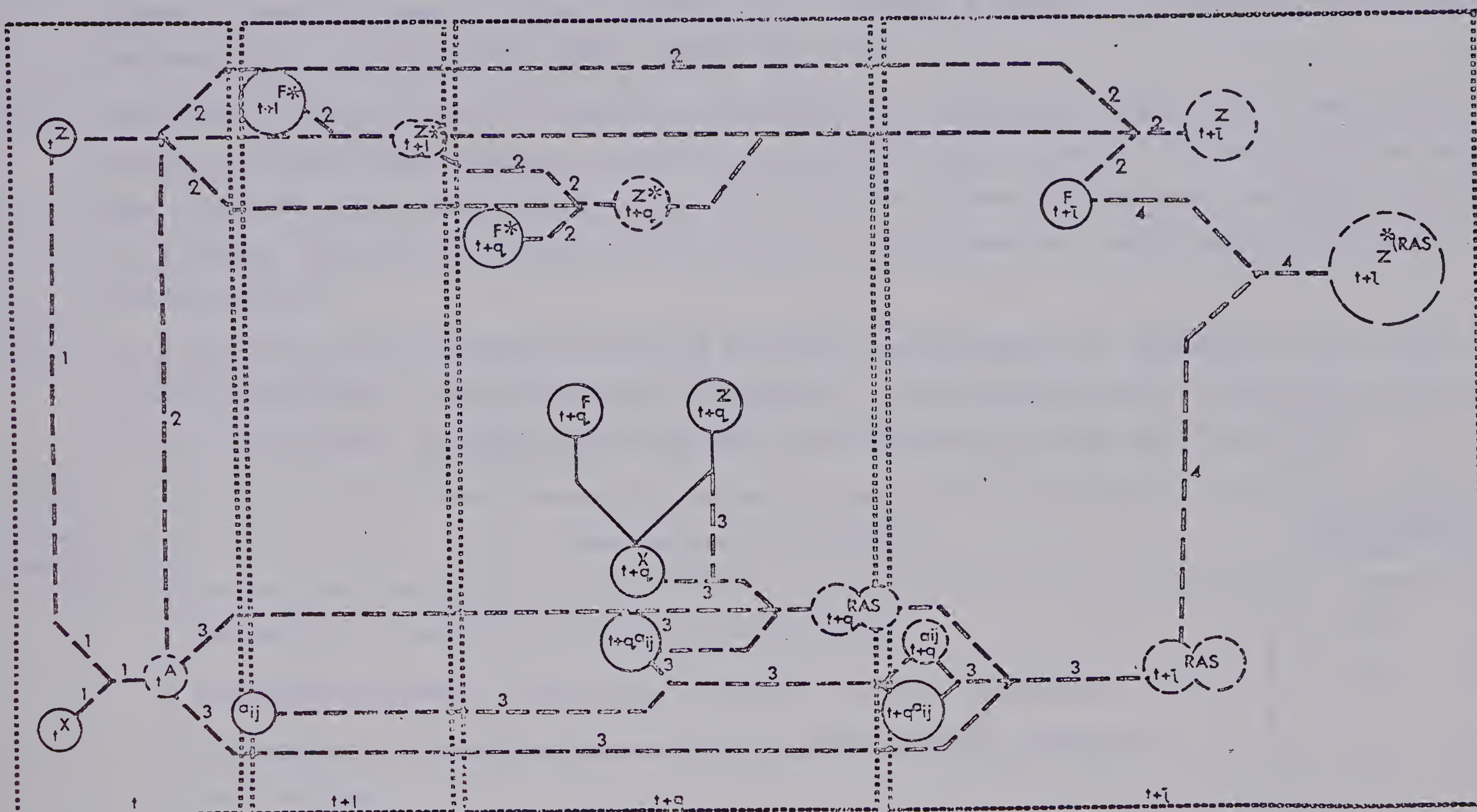
(ii) The circles represent sets of quantities, specified by the symbols inside them. The dotted circles contain estimates derived from the computing sequence itself. The solid circles contain data obtained either from direct observations relating to the past or from direct information on future final demands provided by outside sources.

(iii) The directed lines connecting the circles represent relationships or operations. The solid lines represent identities. Each operation is numbered. A brief description of these operations and a key to the chapter and page where they have been described in full are given below:

No. of operation	Description of operation	Text reference Chapter	Page
1	Estimation of initial input-output technical coefficients	III	13
2	Intermediate demand projections for years $t+1, t+2, \ldots\ldots, t+q, \ldots\ldots, t+\tau$	III	14
3	Estimation of logarithmic prediction errors for intermediate demand in year $t+q$	III V	16 31
4	Revising intermediate demand projections for year $t+\tau$ using the Statistical Correction Method	V	33

Figure 5

A DIAGRAM OF THE COMPUTING SEQUENCE FOR THE RAS METHOD



Notes to Figure 5 Diagram

(i) X = total output; Z = intermediate demand matrix; A = technical coefficients matrix; a = an element or set of elements of the A matrix; RAS = revised A matrix; f = vector of final demands. The subscripts preceding each symbol denote the year.

(ii) The circles represent sets of quantities, specified by the symbols inside them. The dotted circles contain estimates derived from the computing sequence itself. The solid circles contain data. Data in this context may take the form either of direct observations relating to the past or of direct information on future final demands and input-output coefficients provided by outside sources.

(iii) The directed lines connecting the circles represent relationships or operations. The solid lines represent identities. Each operation is numbered. A brief description of these operations and a key to the chapter and page where they have been described in full are given below:

No. of operation	Description of operation	Text reference	
		Chapter	Page
1	Estimation of initial input-output coefficients	III	13
2	Intermediate demand projections for years $t+1, t+2, \ldots, t+q, \ldots\ldots, t+\tau$	III	14
3	Estimation of changes in input-output coefficients and revision of the A matrix	V	34 35
4	Revised intermediate demand projections for year $t+\tau$ using the RAS method	V	36

CHAPTER VII

SUMMARY AND CONCLUSIONS

To the extent that governments and businesses are interested in planning their economic activities ahead of time, economic forecasts are essential. Input-output analysis is an analytical technique that may be usefully employed in obtaining quantitative conditional forecasts of the levels of activities of various interdependent economic sectors. Economic predictions thus obtained are subject to errors, which may be partially reduced by a refinement of the method and material of input-output matrix predictions.

The three-fold objectives of the present study were attained in the order listed in the introductory chapter. Following an investigation of the mathematical properties and the statistical structure of input-output prediction errors, which was the first objective, two methods of revising and improving forecasts were presented in order to attain the second objective. The methods presented were:-

(i) The Statistical Correction Method (The S.C.M.)

(ii) The RAS Method.

Both these methods are so designed as to take into account continuously the experience gained from past attempts at economic forecasting. These methods are also capable of incorporating relevant extraneous information about the operation of the economy and technological and institutional change.

At various stages of the theoretical presentation, it was noted that consideration of some aspects of economic dynamics would be of help

in deriving successful forecasts. With this in view a simplified mathematical model of a Leontief dynamic forecasting system was proposed in the Appendix to Chapter III.

In proposing an application of the theoretical sections of the study to the Province of Alberta, several uses of static and dynamic input-output analysis were cited. Finally, in obtaining the third objective of the study, an inter-industry classification of the economy of Alberta was attempted, paying particular attention to the principal water-using industrial groups.

Conclusions

The input-output model technique of forecasting makes it possible to present alternative forecasts under alternative policy assumptions. It is useful to distinguish conceptually between planning models designed to study and guide the optimum allocation of resources toward some stated objective and models intended as a description of economic behaviour in reality. Input-output analysis belongs to the first category of models.

Once the decision is made to use the input-output frame of references, a statistical reporting system can be established to obtain data on the flow of goods and services. Such a system will eliminate many of the data problems. If an inter-industry table is to be compiled from primary data collected for this purpose, then the organisation of data collection could be so designed that margins of errors be more or less evenly distributed throughout the table; in this way the efficiency of a given amount of information would be maximised.

To the extent that input-output tables and national accounts have

important points in common, the quality of statistics would be improved by a unification of the two sets of accounts, since new possibilities for checking on, and disclosing, inconsistencies would appear. Such unification would also allow an input-output production model to be linked up with some macro-economic models of equilibrium or growth. Promising possibilities for unifying the national accounts and input-output analysis exist in countries where the national accounting systems are fairly well-developed, as they are in Canada, federally and provincially. Careful organisation of the national accounting work here may be quite adequate to obtain an input-output table automatically as a by-product of work normally undertaken by official statisticians in any case. The possibility also exists to draw upon the knowledge of commodity and industrial specialists in a manner not elsewhere possible. A regular flow of accurate and detailed data can also go a long way towards helping to eliminate some of the linearity and constancy assumptions of Leontief models.

In practice some sectors may be found to provide much more detailed quantitative information than can be used in a specific model of the whole economy. A close corollary of such a situation is a situation where some sectors are purposely singled out for detailed analysis within the context of a whole economy. Such a case is well-illustrated by the attempt to emphasise the water-based sectors of the economy of Alberta. The way to obtain more detail is to set up a model-system, with a number of satellite models of industry groups—such as the water supply system; food and beverages manufacturing; services—connected to each other by a central model. Most of the detailed work could then be done by submodels operated by people with specialist knowledge, and the general overall model could be used as an adjusting and co-ordinating mechanism for keeping

the submodels in step.

The implication of considering quantitative economic forecasting in a decision-oriented format is that a set of policies may be recommended that would maximise a welfare function. The maximisation of such a function, in turn, implies that a competent forecaster must know the relevant variables of the welfare function of the particular decision-makers to whom he is supplying his predictions. This study may be ended with the note that the problem of specifying a welfare function in econometrically tractable terms is a difficult one. The problem is more complicated by the intrusion of institutional and socio-psychological factors. The importance of these factors cannot be underestimated even though they fall outside the scope of this study.

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APPENDICES

APPENDIX TO CHAPTER III

III A. Final Demand Projection Techniques

Three categories of projection techniques may be distinguished:-

1. the fitting and extrapolation of straight line or mathematical trends,[1]
2. cyclical or harmonic analysis of the residuals from trend,[1]
3. multi-equational model analysis.

Trend Extrapolation

The simplest approach in trend extrapolation is to treat a component of final demand as a function of time and make future projections on the basis of available time series.

Let y_t = the series to be forecast,

$\phi(t)$ = the function used for the representation of y_t,

ε_t = residual deviation.

Then

$$y_t = \phi(t) + \varepsilon_t \quad , \qquad (A\cdot 1)$$

The specific trend equation adopted may be a simple linear trend, a non-linear trend or a weighted moving average.

In another approach called the leading series method, the series y_t to be forecast is represented as a function of another series x_t, lagged by k time units. Thus if final demand for the products of, say, the agricultural sector depends on consumers' disposable incomes, then

$$y_t = \phi(x_{t-k}) + \varepsilon_t \quad , \qquad (A\cdot 2)$$

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[1] Charles F. Roos, "Survey of Economic Forecasting Techniques," Econometrica, XXIII (October, 1955), 365.

where,

y_t = final demand for the products of the agricultural sector,

x_t = consumers' disposable incomes,

ε_t = residual deviation.

Again different specific functional forms of (A·2) may be adopted, and a number of leading series, such as, product prices, price and income elasticities of demand, and consumers' tastes may be used to project y_t.

Harmonic Analysis of the Residuals from Trend

Harmonic analysis may be used for projection purposes if the removal of trend from the time series of final demand reveals a cyclical, or quasi-cyclical, structure in the resulting residuals. The name, harmonic characteristics, is applied to cyclical features of time series, and the theory of harmonic analysis is applied to calculate the period, intensity, and shape of cycles from historic data. Future values of final demand may then be predicted by a simple projection of the cycles. The harmonic characteristics of a given set of data may be represented as:

$$y = A \text{ Cos } \frac{2\pi t}{T} + B \text{ Sin } \frac{2\pi t}{T} , \qquad (A\cdot 3)$$

or $$y = \sqrt{A^2 + B^2} \text{ Cos } \left(\frac{2\pi t}{T} - a\right)$$

where, y = the series of final demand to be forecast,

A = the amplitude of a cosine wave,

B = the amplitude of a sine wave,

$\sqrt{A^2 + B^2}$ = the amplitude of a combined cosine and sine wave,

T = the period of the harmonic,

$1/T$ = the frequency of the harmonic,

a = arc tan B/A = phase angle,

t = time

π = constant

Multi-equational Model Analysis

An underlying approach in model analysis is the chain principle.[1] Let

$$y_t = \phi(y_{t-k}\ ,\ y_{t-2k}\ ,\ y_{t-3k}\ ,\ \ldots\ldots\ ) + \varepsilon_t \quad , \qquad (A\cdot4)$$

$$k = \text{lag unit}\ ;\quad \varepsilon_t = \text{residual disturbance},$$

be a model designed to forecast final demand, y_t , on the basis of past observations, y_{t-k} , y_{t-2k} , ... The chain principle then uses the forecast value of y_t as an actual observation, y^*_t , and obtains the forecasts y^*_{t+1} , y^*_{t+2k} , ... by a chain of iterative substitutions. The resulting forecasts are

$$\begin{aligned} y^*_t &= \phi(y_{t-k}\ ,\ y_{t-2k}\ ,\ y_{t-3k}\ ,\ \ldots\ldots\ ) \\ y^*_{t+k} &= \phi(y^*_t\ ,\ y_{t-k}\ ,\ y_{t-2k}\ ,\ \ldots\ldots\ ) \\ y^*_{t+2k} &= \phi(y^*_{t+k}\ ,\ y^*_t\ ,\ y_{t-k}\ ,\ \ldots\ldots\ ) \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \end{aligned} \quad , \qquad (A\cdot5)$$

In a multi-equational model the variable y_t to be forecast is a vector of several component variables. Thus

$$y_{i_t} = \phi_i\ (y_{i_{t-k}}\ ,\ y_{i_{t-2k}}\ ,\ y_{i_{t-3k}}\ ,\ \ldots\ldots) + \varepsilon_{i_t}\ ;\ i = 1\ ,\ \ldots,\ m\ , \quad (A\cdot6)$$

In addition, the model may be a multi-relation one, incorporating a vector, x_t , of several other exogenous variables:

$$y_{i_t} = \phi_i\ (y_{t-k}\ ,\ y_{t-2k}\ ,\ y_{t-3k}\ ;\ x_t\ ,\ x_{t-k}\ ,\ x_{t-2k}\ ,\ \ldots.) + \varepsilon_{i_t} \quad , (A\cdot7)$$

where,

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[1] Herman O. A. Wold, editor, Econometric Model Building (Amsterdam: North-Holland Publishing Company, 1964), p. 8.

$$i = 1 , \ldots , m ,$$

$$y_t = (y_{1t} , y_{2t} , \ldots\ldots , y_{mt}) ,$$

$$x_t = (x_{1t} , x_{2t} , \ldots\ldots , x_{ht}) .$$

A multi-equational model is useful for the simultaneous projection of whole vectors of the components of final demand.

III B. Adjusting Input-Output Predictions for Price Changes

Conventional input-output prediction theory deals with physical volumes, whereas many predictions are made in current prices. Hence input-output tables of technical coefficients may have to be adjusted for price changes.

If it is assumed that for each sector one price index is applicable to all components of the sector's output, then the input coefficient $a_{ij} = Z_{ij}/X_j$ may be replaced by

$$\frac{P_j}{P_i} a_{ij} = \frac{Z_{ij}/P_i}{X_j/P_j} , \qquad (B\cdot1)$$

in order to use input coefficients in constant prices:

P_i is the price index of the output of sector i used to deflate the sector's sales to sector j,

P_j is the price index of the output of sector j used to deflate the sector's total output.

Further, let P be a diagonal matrix whose (i,i)-th element is p_i; then $p_j a_{ij}/p_i$ is the (i,j)-th element of the matrix $P^{-1}AP$. Now in using the input-output table of year t for prediction purposes the matrix multiplier $(I - A_t)^{-1}$ is replaced by

$$(I - P_t^{-1}A_tP_t)^{-1} = \left[P_t^{-1} (I - A_t)P_t\right]^{-1} = P_t^{-1}(I - A_t)^{-1}P_t , \qquad (B\cdot2)$$

The final demand vector in the prediction year in constant prices would be written $P^{-1}_{t+\tau} f_{t+\tau}$, which may be multiplied by

$$P^{-1}_{t}(I - A_{t})^{-1}P_{t} - I = P^{-1}_{t}\left[(I - A_{t})^{-1} - I\right]P_{t} \quad , \qquad (B\cdot 3)$$

in order to obtain the prediction of the intermediate demand vector $Z^{*}_{t\tau}$. Hence given the final demand and the prices of year $t + \tau$,

$$Z^{*}_{t\tau} = P^{-1}_{t}\left[(I - A_{t})^{-1} - I\right]P_{t}\, P^{-1}_{t+\tau} f_{t+\tau} \quad , \qquad (B\cdot 4)$$

Taking the year t as the base for the various price indices, so that $P_{t} = I$, (B·4) expresses intermediate demand in the prices of year t. To obtain a forecast in the prices of year $t + \tau$, premultiply (B·4) by $P_{t+\tau}$, so that

$$Z^{*\text{in } t+\tau \text{ prices}}_{t\tau} = P_{t+\tau}\, P^{-1}_{t}\left[(I - A_{t})^{-1} - I\right]P_{t}\, P^{-1}_{t+\tau} f_{t+\tau} \quad , \qquad (B\cdot 5)$$

III C. Adjusting Input-Output Coefficients for Technological Changes

The term technological change here is used to refer to those changes in the economy that affect the technical coefficients of an input-output table. Three principal types of changes may be distinguished: changes in relative prices, the appearance of new industries, and technological changes proper.

Changes in relative prices of factors of production during the forecast period may cause input patterns, hence some of the technical coefficients, to change. This latter change will occur only if some inputs can be substituted for others. Changes in technical coefficients as a result of price-induced substitutions are more pronounced the more disaggregated the tables are.

The appearance of one or more new industries during the prediction period might partially invalidate a long-term consistent forecast. Therefore, earlier forecasts have to be adjusted to take into account new forms of economic activity.

Technological changes proper refer to changes in factor productivities, which may be due to technical and organisational innovations.

The nature and rates of change of the various components of an input-output matrix may be taken into account in a dynamic input-output forecasting model. One or more of the following dynamic characteristics may be included in any single model.[1]

1. The inclusion of trends in the coefficients of some rows of the technical matrix,

2. Assumptions about the course of foreign trade and net foreign balances,

3. Assumptions about government expenditure and demand,

4. The specification of a consumption function,

5. The phasing of investment and capital accumulation equations. "Phased equations" connect the different variables from one projection period to another in some specified order.

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[1]Clopper Almon, Jr., The American Economy to 1975 (New York: Harper and Row, Publishers, 1966), pp. 141-146; Anne P. Carter, "Incremental Flow Coefficients for a Dynamic Input-Output Model with Changing Technology," Structural Interdependence and Economic Development, edited by Tibor Barna (New York: St. Martin's Press, 1963); Robert M. Solow, "Investment and Technical Progress," Mathematical Methods in the Social Sciences, edited by Kenneth Arrow et al. (Stanford: Stanford University Press, 1960); Clopper Abman, "Consistent Forecasting in a Dynamic Multi-Sector Model," Review of Economics and Statistics, XLV (May, 1963), 148-161; Almon, "Numerical Solution of a Modified Leontief Dynamic System for Consistent Forecasting or Indicative Planning," Econometrica, XXXI, No. 4 (October, 1963), 665-678.

In the following model, the matrix of flow coefficients, A, is assumed to be linearly determined by time:

$$A_t = A_1 + A_2(t) \quad , \tag{C·1}$$

where, A_1 and A_2 are constant matrices, known in advance,

t represents time.

Government purchases and exports are considered to be known coefficients of time.

Consumer demand may be determined by multiplying the number of consumers, which is a function of time, by the consumption per consumer, which depends upon w , an index of the real wage rate. Thus the vector of final demands, f , depends on w and t, time:

$$f = \phi(t\ , w) \quad , \tag{C·2}$$

Assuming linear trend for exports, government spending, and the number of consumers and assuming the per capita consumption of each commodity to be a linear function—with a positive intercept—of the real wage rate, the final demand function may take the following form:

$$f = \phi_1 + \phi_2 t + (w(t) - 1)\ (\phi_3 + \phi_4 t) \quad , \tag{C·3}$$

where,

ϕ_1 , ... , ϕ_4 are constant vectors and $w(o) = 1$.

Investment and capital expenditure is made dependent on total output, the wage rate and the rate of increase of factor productivity. Specifically investment results not only from the growth of output, but also from the substitution of capital for labour as wages go up and as the productivity of new capital increases. Substitution of capital for labour is allowed by a Cobb-Douglas production function in each industry. The functions

allow the productivity of new capital to increase exponentially. Thus output of industry j , X_j , is related to capital, K_j , and employment, N_j, in the industry as follows:

$$X_j = \alpha_j N_j^{a_j} K_j^{1-a_j} \quad , \qquad (C\cdot4)$$

where,

α_j and a_j are constants.

The series for capital in time period t is, for any industry,

$$K(t) = \int_{-\infty}^{t} e^{v\tau} \cdot \delta(t-\tau)\, I(\tau)d\tau \ , -\infty \leq \tau \leq t$$

and

$$\dot{K} = \frac{dK(t)}{dt} = e^{vt}\, I(t) - \delta K \quad , \qquad (C\cdot5)$$

where, v = the rate of increase in the productivity of capital,

δ = the rate of depreciation of capital,

$I(t)$ = the series of gross investment in constant dollars.

The increase in the productivity of capital may be interpreted as a decline, at the same rate v, in the constant-dollar price of a unit of capital. Assuming a fixed cost of funds, r, and setting r = 1 at t = o, the price of capital at time t = o is e^{-vt}. In determining the relation between I(t), w, and v it is assumed that industrial sectors minimise the sum of labour and capital costs,

$$w(t)N_j(t) + r\, e^{-vt} K_j(t)$$

subject to the constraint

$$X_j = \alpha_j N_j^{a_j} K_j^{1-a_j}$$

The Lagrangian expression for the minimisation of the above constrained cost function is:

$$L = W(t)N_j(t) + r\, e^{-vt}K_j(t) - \lambda(t)\left[X_j(t) - \alpha_j N_j(t)^{a_j} K_j(t)^{1-a_j}\right] \quad , \qquad (C\cdot 6)$$

Taking the partials $\frac{\partial L}{\partial N_j} = o$ and $\frac{\partial L}{\partial K_j} = o$, using these to eliminate λ, and then using (C·4) to eliminate N_j, the desired, full-employment, level of capital in industry j at time t is found to be:

$$K_j(t) = C_j X_j(t) w(t)^{a_j} e^{a_j v_j t} \quad , \qquad (C\cdot 7)$$

where C_j is the full-employment capital-output ratio at t = o. Differentiating (C·7) with respect to time to obtain $\dot{K}_j = \frac{dK_j(t)}{dt}$, and using (C·5) and (C·7) to eliminate $\dot{K}_j$ and K_j, respectively, gives

$$I_j(t) = C_j w^{a_j} e^{-(1-a_j)\, v_j t}\left[\dot{X}_j + a_j \frac{\dot{w}}{w} (a_j v_j + \delta_j)\right] X_j \quad , \qquad (C\cdot 8)$$

Equation (C·8) gives the total amount of expenditure by the j-th industry in constant dollars.

The vector of total investment demands for the output of each industry is

$$I(t) = BG_1(t,w)X(t) + BG_2(t,w)\, DX(t) \quad , \qquad (C\cdot 9)$$

where,

$G_1(t,w)$ is a diagonal matrix with the j-th diagonal element being

$$e^{-(1-a_j)v_j t}\left[a_j w^{-(1-a_j)}\, \dot{w} + (a_j v_j + \delta_j) w^{a_j}\right] \quad ,$$

$G_2(t,w)$ is a diagonal matrix with the j-th diagonal element being

$$e^{-(1-a_j)v_j t}\, w^{a_j} \quad ,$$

$\mathcal{B}$ is the matrix showing how much of the produce of industry i is needed as capital by industry j for a unit increase in the output of j.

$X(t)$ is a vector of outputs.

$\mathcal{D}$ is the differential operator.

Hence the fundamental system of equations which the dynamic forecasting model must solve is of the form

$$\underbrace{X(t)}_{\text{TOTAL OUTPUT}} = \underbrace{A(t)X(t)}_{\text{INTERMEDIATE DEMAND}} + \underbrace{BG_1(t,w)X(t) + BG_2(t,w)\ \mathcal{D}X(t)}_{\text{DYNAMIC INVESTMENT DEMAND}} + \underbrace{f(t,w)}_{\text{FINAL DEMAND}} \qquad (C\cdot 10)$$

APPENDIX TO CHAPTER VI

VI A. Input-Output Analysis and The Optimum Allocation of Resources

A search for an optimum or efficient mode of production necessarily implies a choice among alternative techniques of production. Now a strictly Leontief-type input-output model, with its characteristic square matrix of technical coefficients, does not allow for choice of techniques. However, only a slight modification of the Leontief model is required to introduce choice. What is needed is a rectangular instead of a square matrix of coefficients. The desired rectangularity may be obtained by introducing more columns in the coefficients matrix than rows; that is, by including more ways of producing things than things to be produced. Choice among alternative techniques can then be directed towards a full employment of all scarce factors and to maximising the composite output in the sense of seeking a position in which any further increase in the output of one commodity necessitates a decrease in the output of at least one other commodity.[1] The technological ratios that render such an optimum position feasible may then be interpreted as imputed relative prices and used as a guide to the optimum allocation and use of available resources.

VI B. A Dynamic Input-Output Model

A static input-output model of the Province of Alberta would be formulated solely in terms of flows of commodities and services. Using

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[1]Nicholas Georgescu-Roegen, "The Aggregate Linear Production Function and Its Applications to von Neumann's Economic Model," Activity Analysis of Production and Allocation (New York: John Wiley and Sons, 1965), pp. 98-115; especially pp. 98-108.

the same basic format, the distribution of all kinds of stocks among the different sectors could be summarised for a selected base year. These stock figures would include inventories of raw materials, semi-finished and finished products, stocks of fixed plant and equipment, residential dwellings, and stocks of consumer durables.[1] The static input coefficients would then be supplemented by a corresponding set of stock : output ratios, the amount of stocks held being assumed to be specifically related to the rates of output of corresponding sectors.

The economic structural equations then would contain two sets of structural constants: the current input ratios, and the stock coefficients. One would thus have a set of linear first order differential equations [(C·10), Appendix to Chapter III]. The significance of these equations is that they contain not only rates of flow of various goods and services but also the rates of changes of the rates of flow.[2] For predictive purposes the final demand can be assumed to exhibit a changing pattern over the forecast period [(C·3), Appendix to Chapter III], thus enabling the determination of the necessary—and changing—pattern of output of the intermediate goods and services over the same forecast period [(C·9), Appendix to Chapter III]. Forecasts obtained in this way would include numerical indications of the changing levels of stocks, investments, disinvestments, and spare capacities.

If a detailed breakdown of stocks and inventories by classified

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[1]Thomson M. Whitin, The Theory of Inventory Management (Princeton: Princeton University Press, 1953).

[2]Julian L. Holley, "A Dynamic Model: I. Principles of Model Structure," Econometrica, XX (October, 1952), 616-642.

types is not available, one might try to obtain single values of the composite amounts of stocks held by each sector. These values may then be inserted in a stock row or column of the open-ended input-output table. Such aggregated stock figures, however, may be objected to on the grounds that they obviously add to the aggregation problem already present in any industrial classificatory system.

VI C. The Proposed Classification for an Input-Output Transactions Matrix

Intermediate Use Sectors

01 Agriculture
02 Forestry
03 Grain mills
04 Dairy products
05 Slaughtering and meat processing
06 Beverage manufacturing
07 Miscellaneous food industries
08 Mining: oil, gas and non-metallic
09 Primary metal and metal fabricating
10 Machinery and transportation equipment
11 Leather, textiles and clothing
12 Wood products
13 Furniture and fixtures
14 Paper
15 Print-publishing
16 Non-metallic mineral products
17 Petroleum and coal products
18 Chemical products
19 Miscellaneous manufacturing
20 Construction
21 Transport, communication, and storage
22 Electricity

23 Water

24 Trade: Wholesale and retail

25 Banking and finance

26 Real estate and insurance

27 Education, health, and welfare (includes religious organisations, laundries, and cleaners)

28 Recreational services (includes motion pictures, hotels, and restaurants)

29 Public administration

30 Miscellaneous services

<u>Primary Input Sectors</u>

31 Imports

32 Tax, rent, interest

33 Profits

34 Depreciation

35 Wages and Salaries

36 Unallocated input[1]

<u>Final Demand Sectors</u>

01 Household consumption

02 Government expenditure on goods and services

03 Government gross fixed investment

04 Private gross fixed investment

05 Exports

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[1]Unallocated input will represent a positive or negative balancing item where data on total output of an industry do not tally with allocated total inputs.